TOPOLOGY — the mathematics of spaces

Classical Question: Can  be deformed

sphere

into  ?

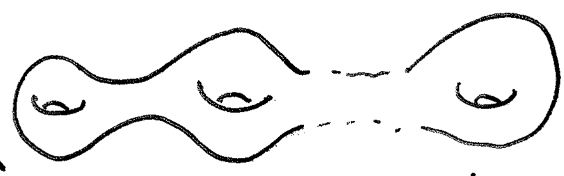
donut

Answer: No. But how are we sure?

Because the "Euler Characteristic χ" detects:

$\chi($  $) = 2 \neq 0 = \chi($  $)$

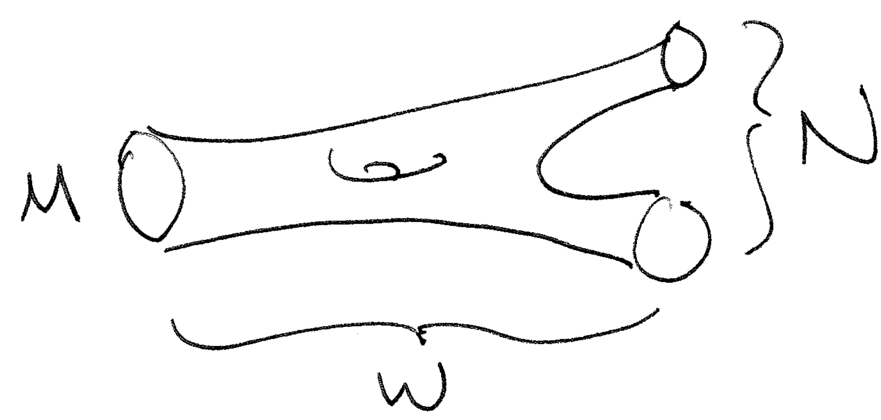
In fact, χ classifies all (compact, oriented) 2-dimensional surfaces:

$\chi($  $) = 2 - 2g$

g holes

Higher-dimensional surfaces are much more difficult to classify.

I am studying the "signature" σ.
It is a "cobordism invariant." (χ is not.)
This means that if two spaces M and N "cobound" a space W:



then $\sigma(M) = \sigma(N)$.

Your pair of pants, for example, say that

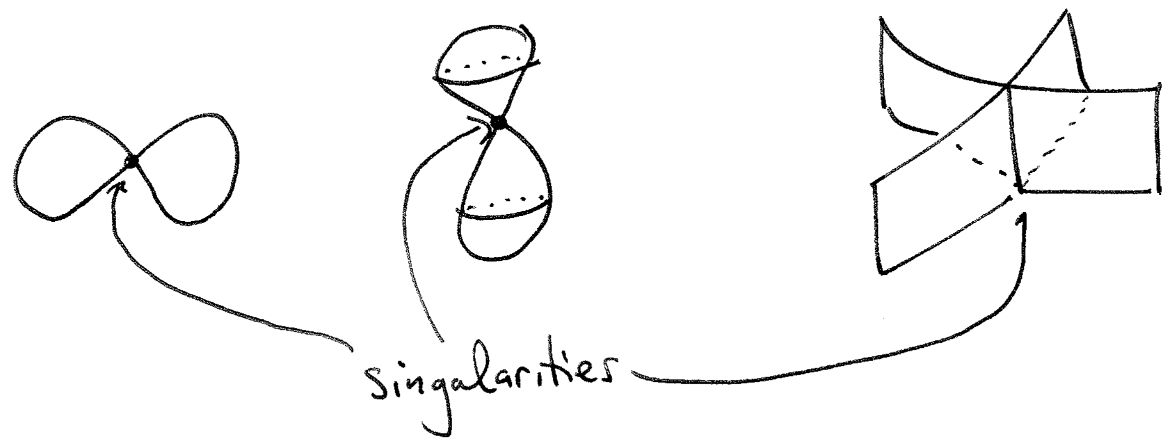
$$\sigma(\text{circle}) = \sigma(2 \text{ circles}) = 2 \cdot \sigma(\text{circle})$$

$$\underset{\text{implies}}{\Longrightarrow} \quad \sigma(\text{circle}) = 0$$

Cobordism provides a steping stone in classifying spaces: first classify "up to cobordism", then refine the classification.

This is all old news. What am I studying?

The classification of spaces with singularities.



In particular, I am studying how to compute the signature σ of such spaces.
(It is surprisingly difficult and involves the "derived category of sheaf complexes".)

The hope is to then use this to classify singular spaces — at least "up to cobordism".

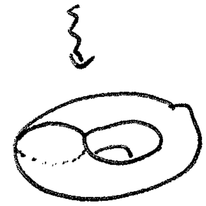
What is the Euler Characteristic χ?

It's the "alternating sum" of the number of "cells" in a space:

0-cells 1-cells 2-cells

$+1 \quad -0 \quad +1 = 2 = \chi($  $)$

$+1 \quad -2 \quad +1$



$= 0 = \chi($  $)$