Music Through Computation

Carl McTague

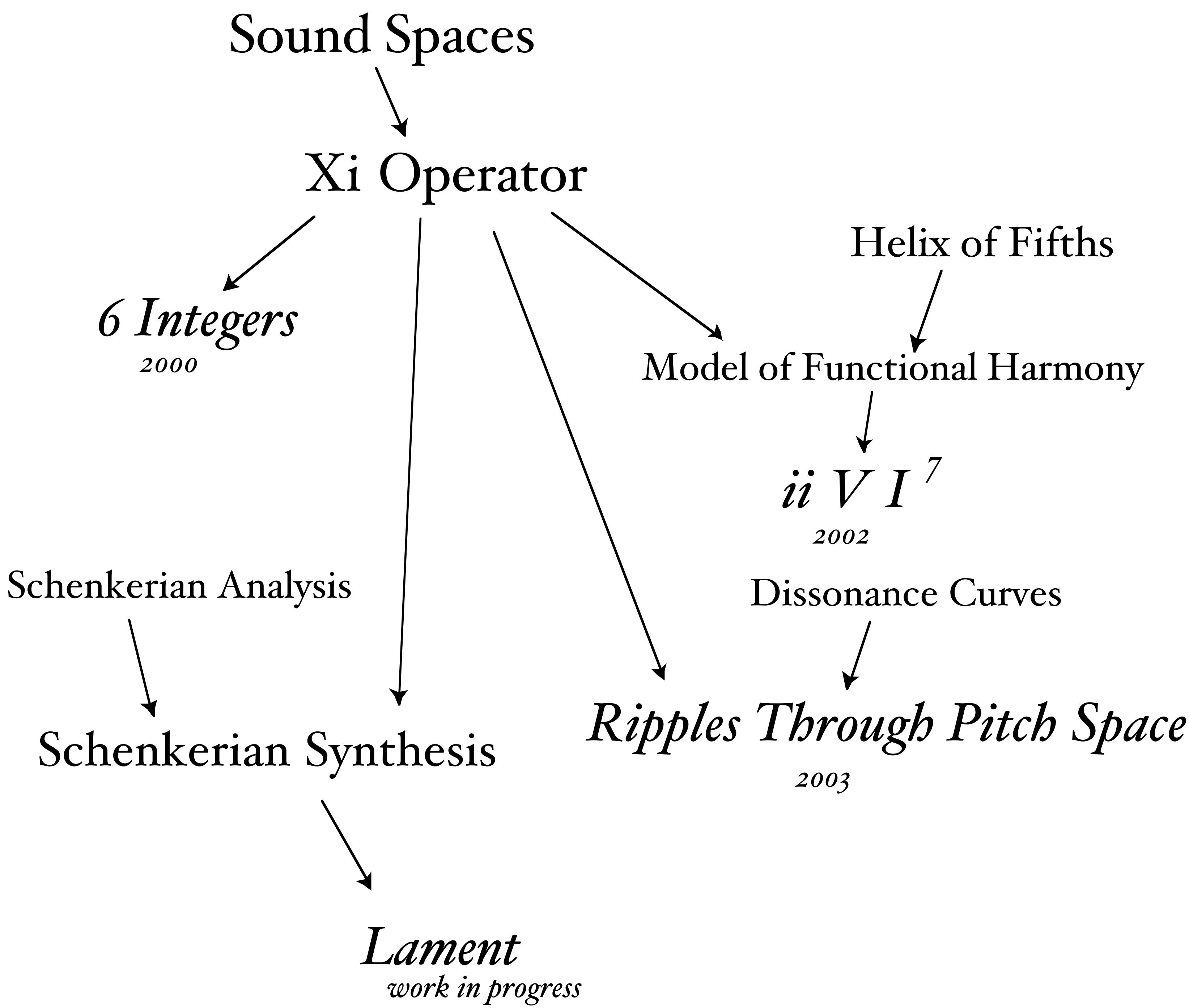
July 7, 2003

International Mathematica Symposium

Objective:

To develop powerful mathematical structures in order to *compose interesting new music*.

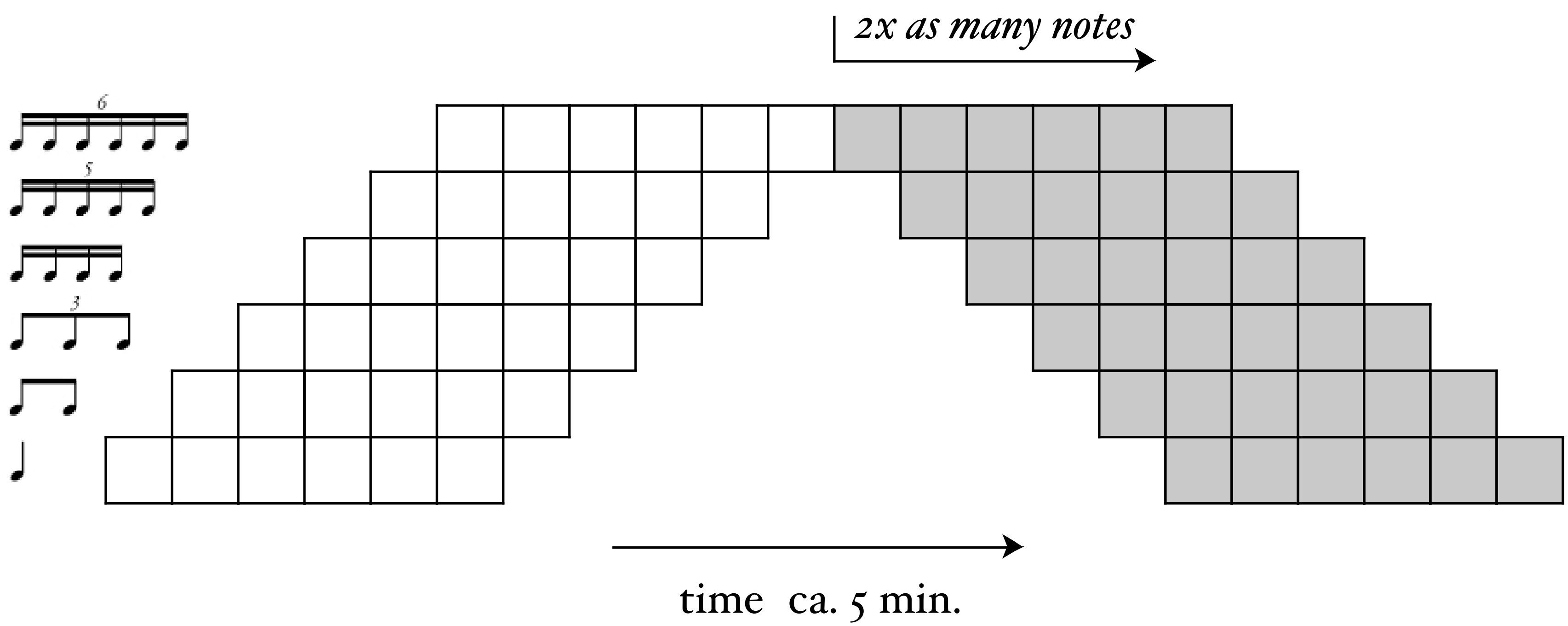
not to analyze existing music   although inspiration often comes from existing music and analytical techniques



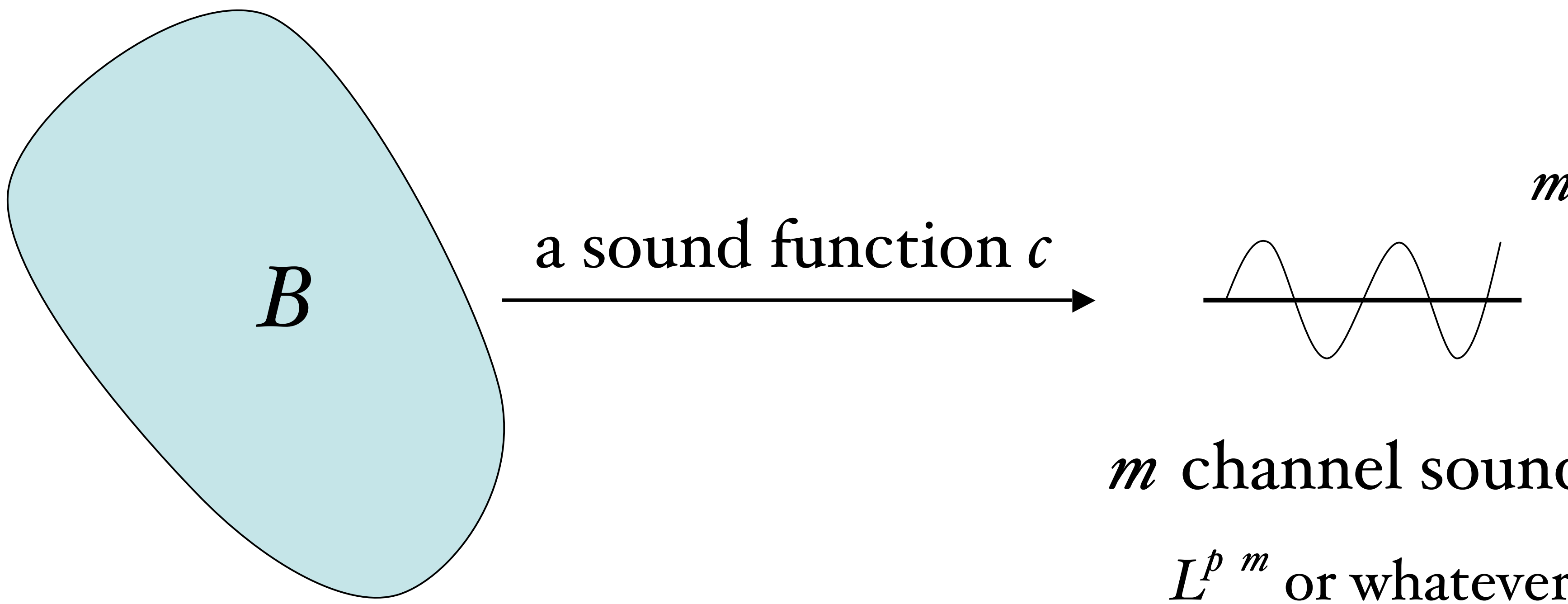
Sound Spaces
Xi Operator
6 Integers
2000
Helix of Fifths
Model of Functional Harmony
ii V I 7
2002
Schenkerian Analysis
Dissonance Curves
Schenkerian Synthesis
Ripples Through Pitch Space
2003
Lament
work in progress

6 Integers

2000



Sound Spaces



m channel sounds

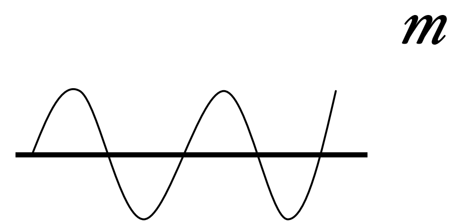
$L^{p\ m}$ or whatever

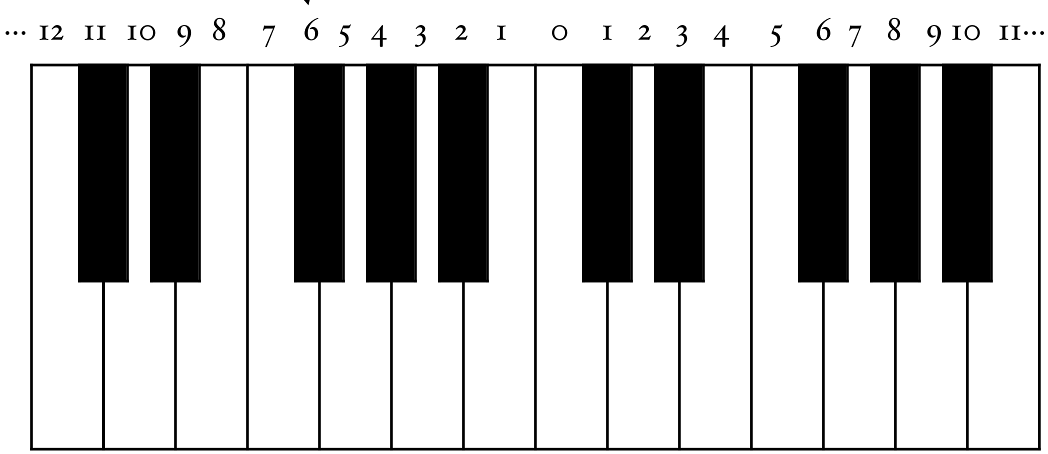
a sound space B

Example: Piano

intensity attack time duration

$$B = Z \prod R^{+} \prod R \prod R \xrightarrow{\;\; piano \;\;}$$





But why bother with sound spaces at all?

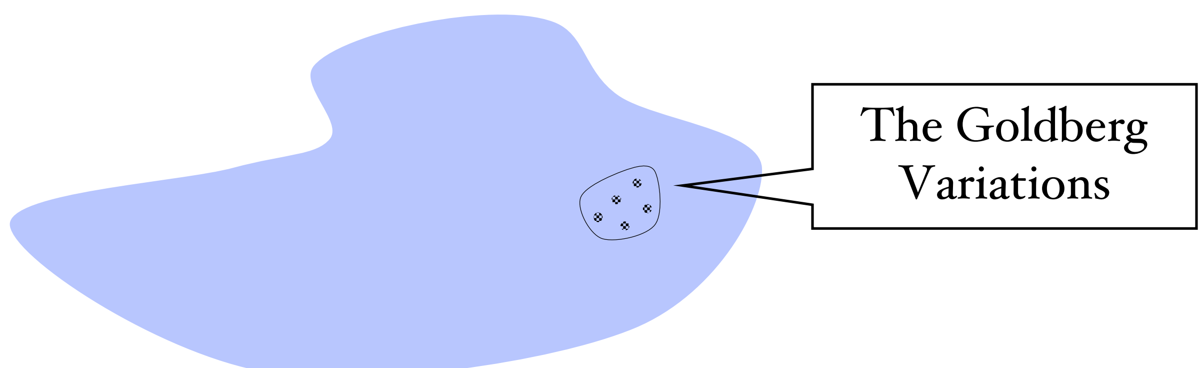
Why not just work directly within $L^{p \quad m}$ or whatever ?

Why use sound spaces?

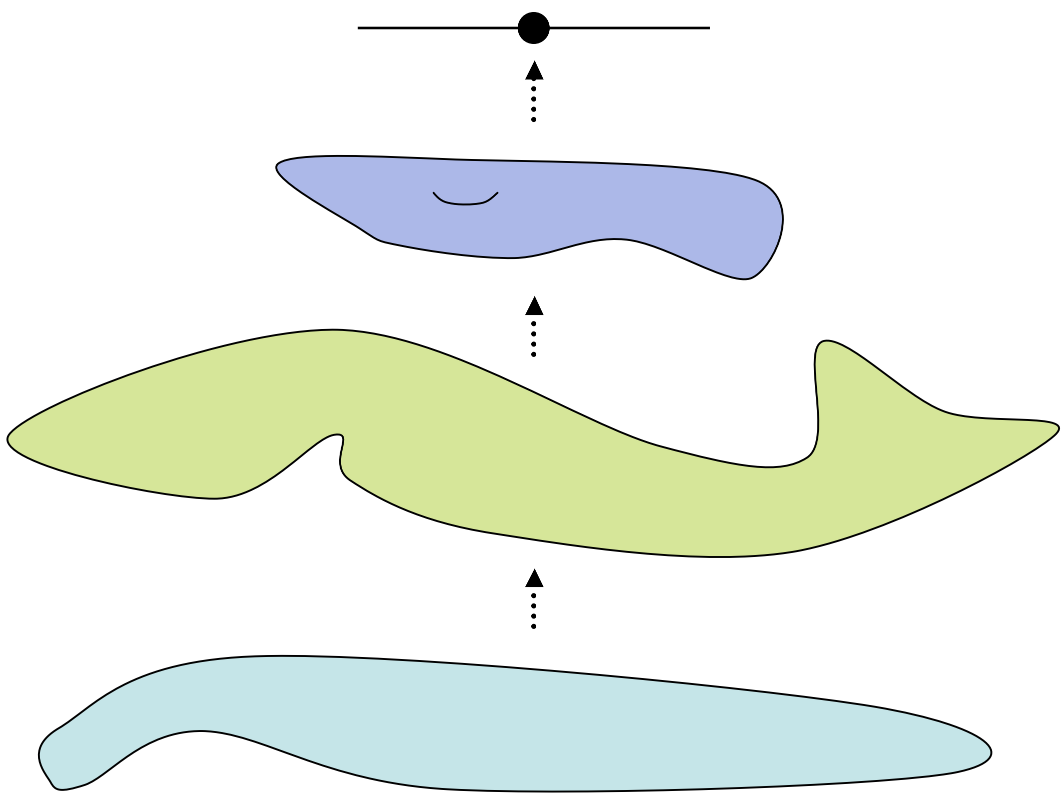
- $L^{p\ \ m}$ is dauntingly **HUGE**! Want to avoid the ultimate writer's block *how do you ever get started in the "space of all possible sounds"?*
- Want nice little representations of sounds inside the computer.

Why use sound spaces?

- Want "musical topologies!" the standard metric on $L^{p}\ {}^{m}$ is too rigid, unmusical.
- Natural e.g. continuous operations on the spaces should correspond to musical processes.
- E.g. variations might lie within neighborhoods:



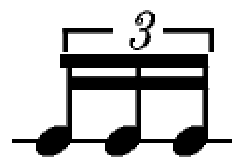
General approach to composition

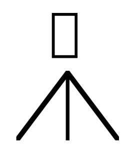


Inductively construct increasingly complex and specialized sound spaces until an entire piece of music is the image of a single, conspicuous point.

Think of it as building increasingly powerful musical instruments.

Simple motivating example:





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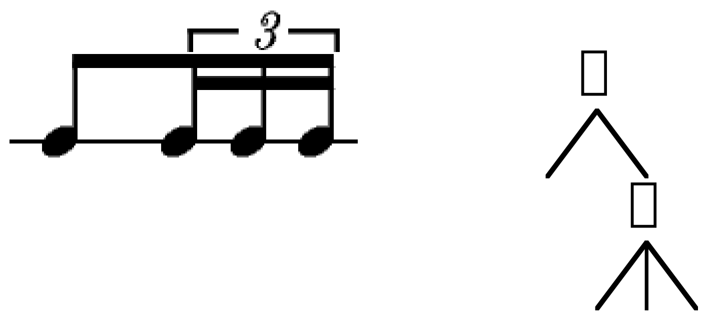

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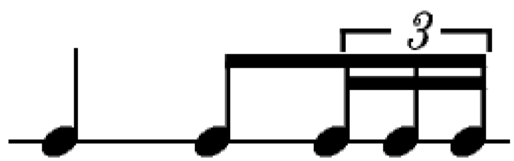

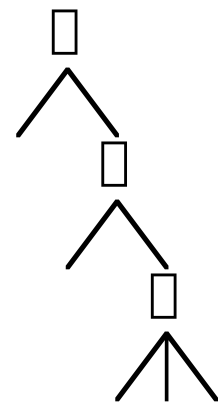

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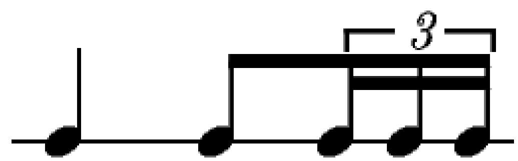
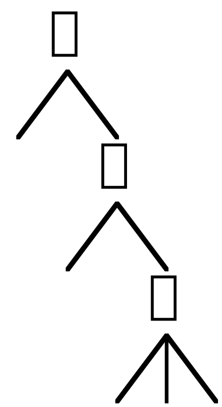
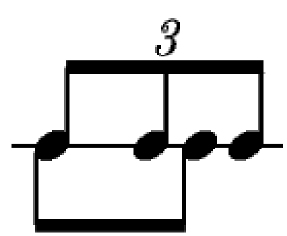
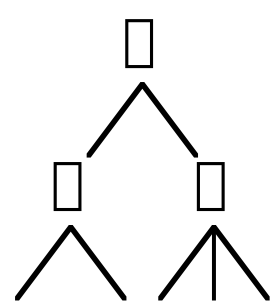






the "Brahms rhythm"



The General Xi Ξ *Construction*

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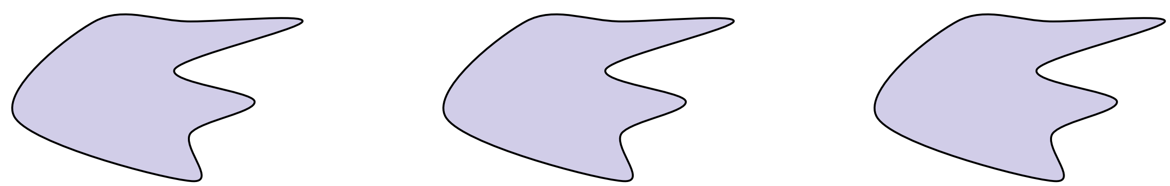
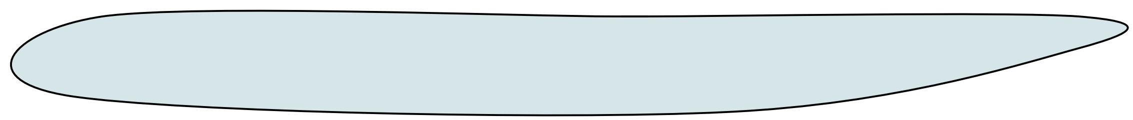
copies of an existing
sound space B
each equipped with a
potentially distinct sound
function



The General Xi Ξ *Construction*

a new space A



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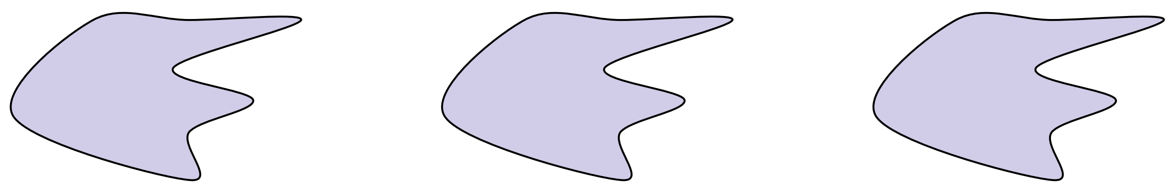

The General Xi Ξ *Construction*

a new space A

a new
"inheritance function"
$f_3 : A \to B^3$

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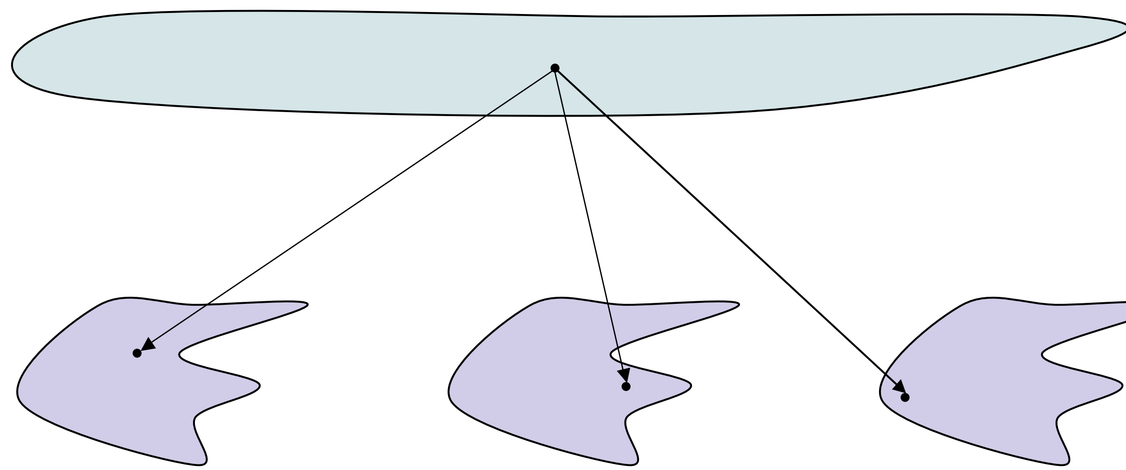

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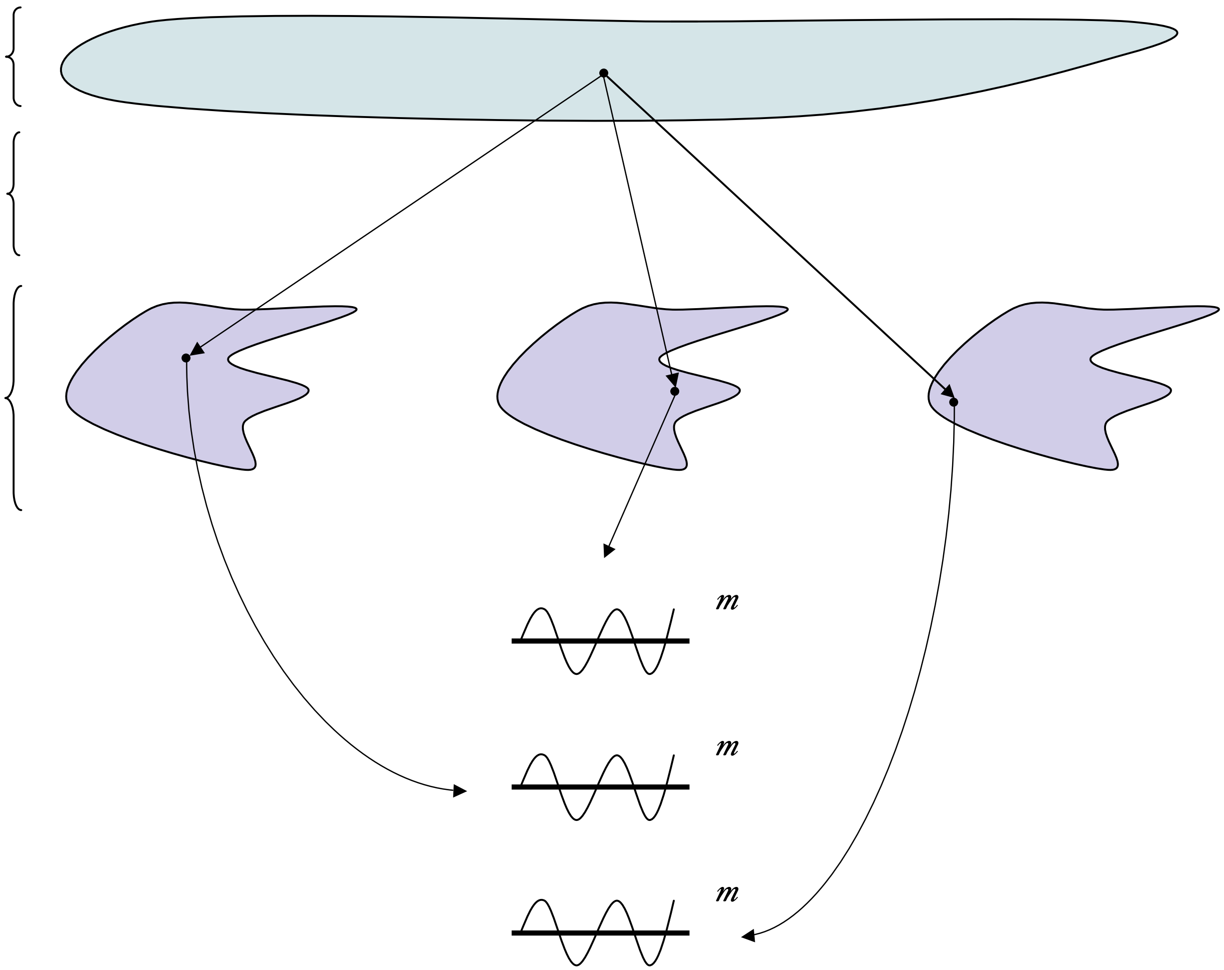

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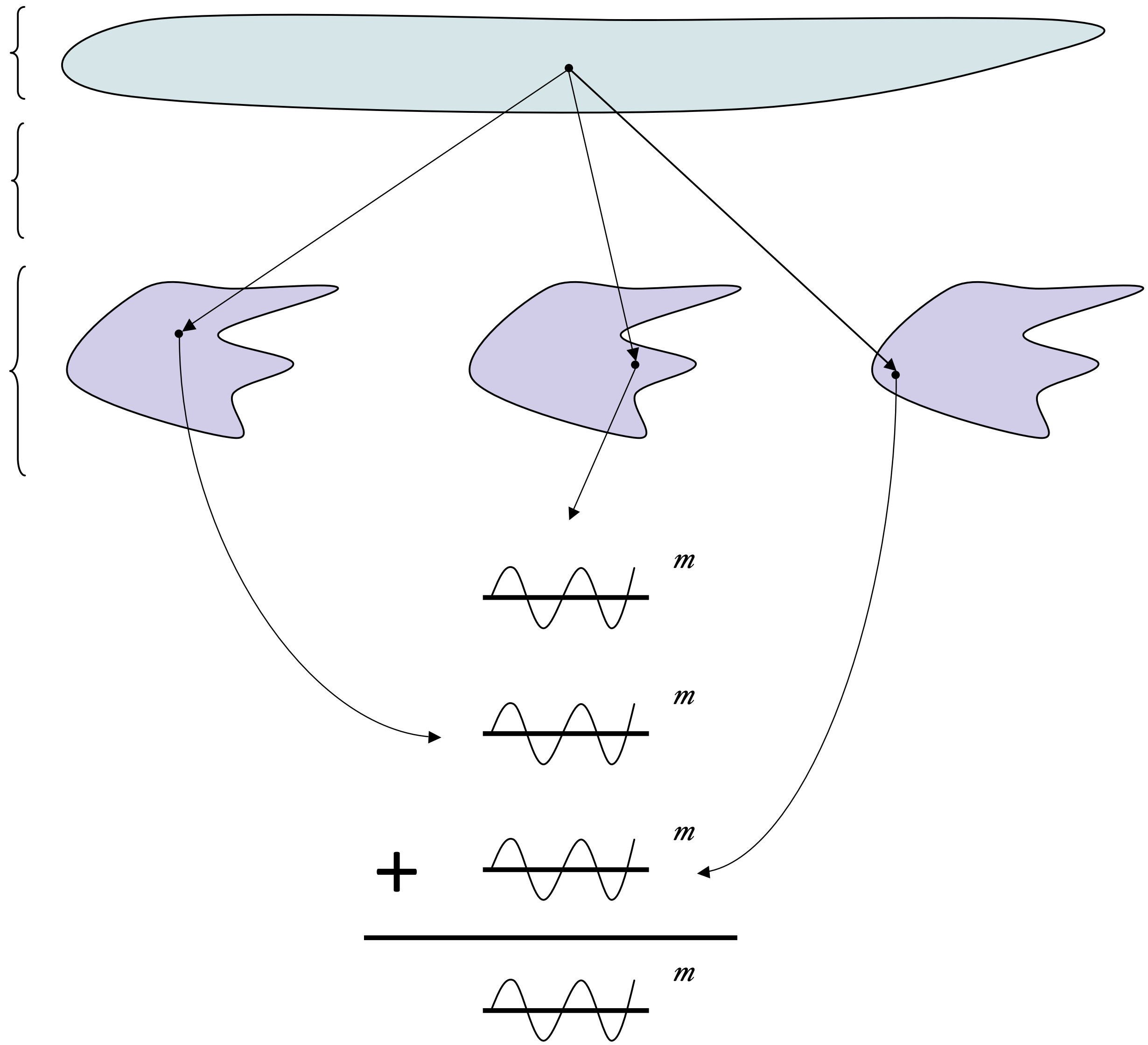

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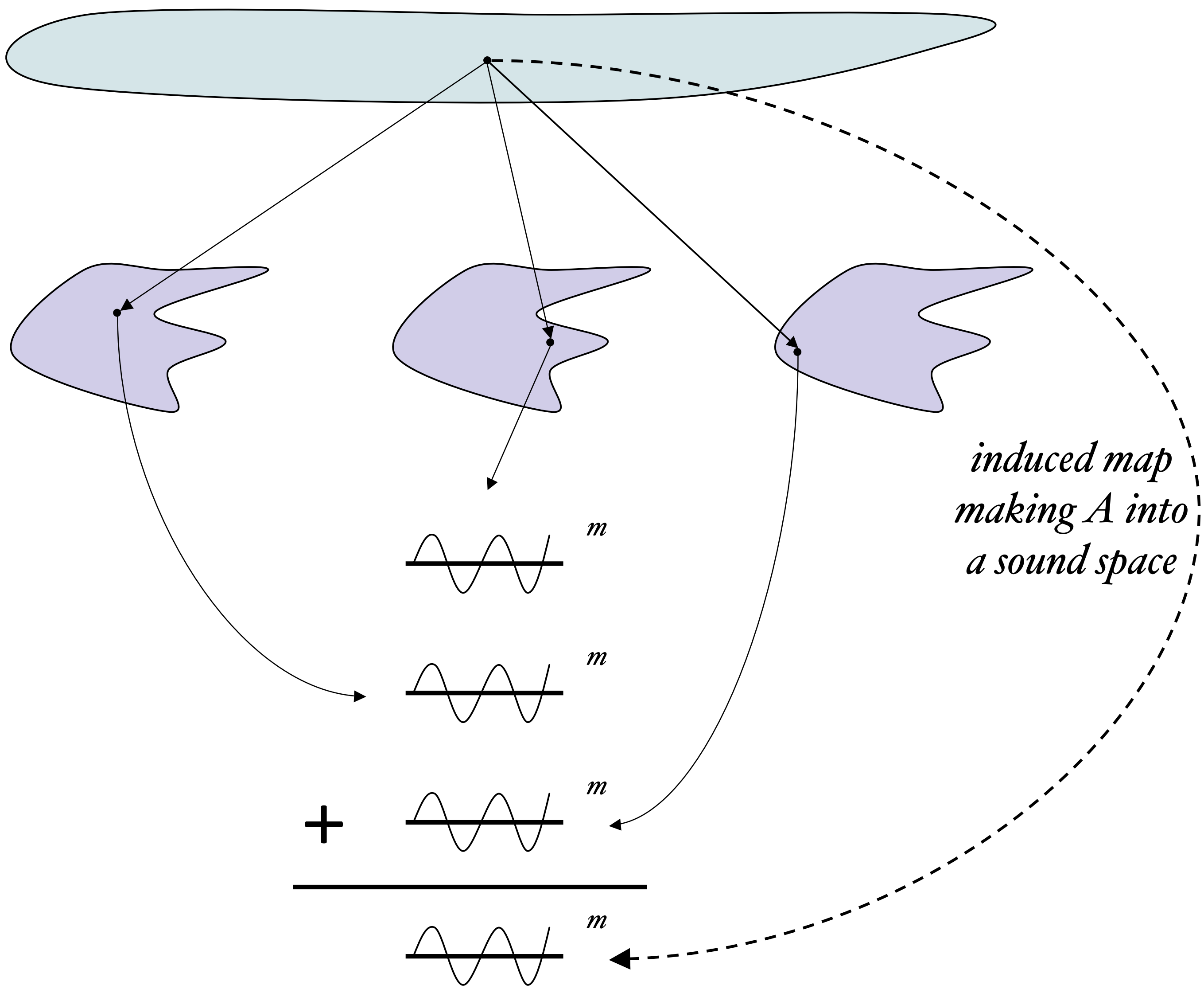
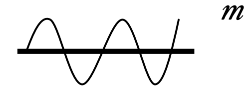
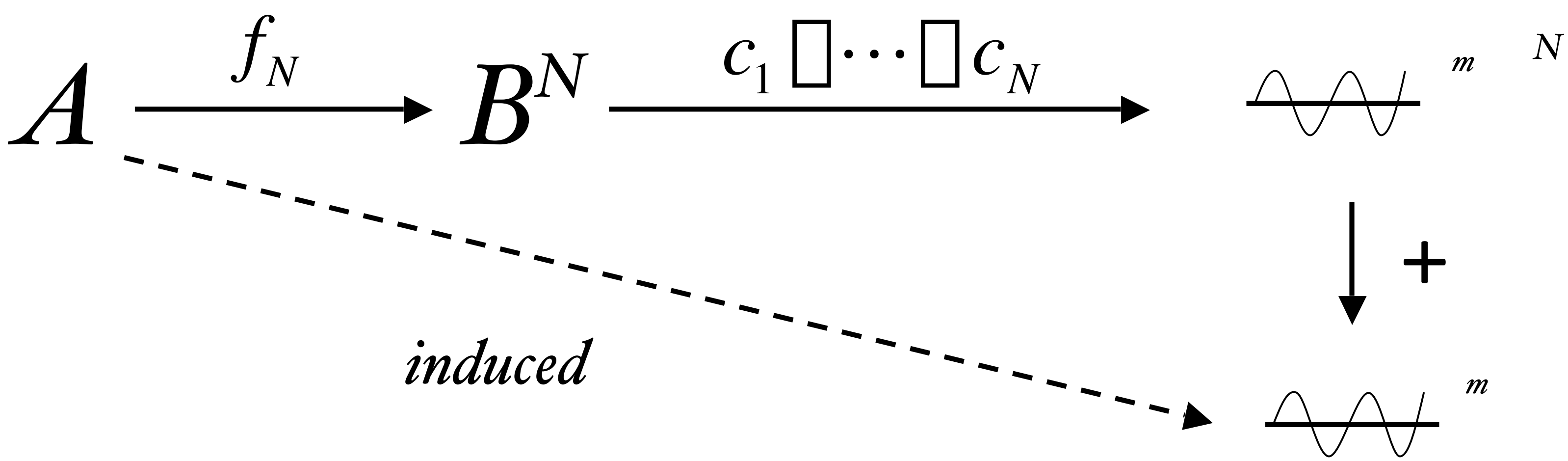


Xi for diagram chasers:

Given a list of sound functions $c_i : B \longrightarrow$  $_{i=1,\dots N}$

and a family of "inheritance functions" $\{f_n : A \longrightarrow B^n\}_n$

make A into a sound space via the induced map:



So, with Xi in hand, we can build new sound spaces by constructing a few:

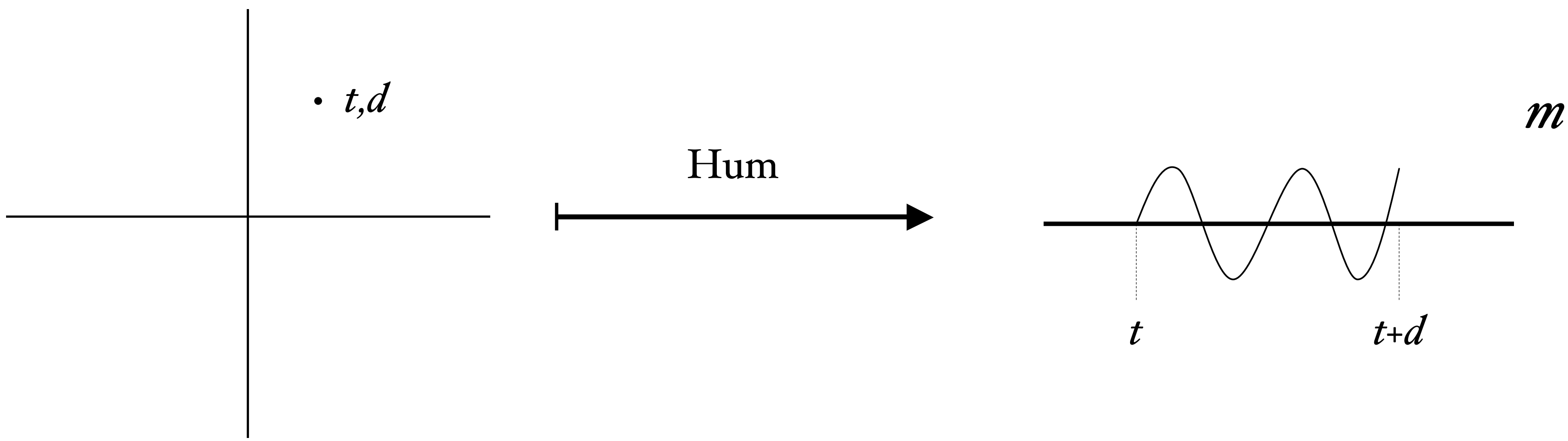
- fundamental sound spaces
- families of inheritance functions

and arranging them into hierarchies.

This is precisely what we do, next…

A simple sound space:

Consider the plane $\mathbb{R}^2$ as a sound space by regarding the point t,d as a hum at time t with duration d.



The so called time vector approach.

Two Useful Families of Inheritance Functions:

The “diagonal maps” for simultaneity:

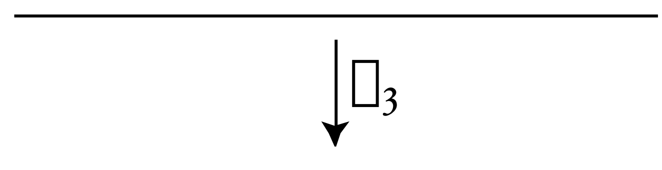
$$A \xrightarrow{\Delta_n} A^n$$

$$a \longmapsto a,\ldots,a$$

just make n copies

For successiveness:

$$\mathbb{R}^2 \xrightarrow{\square_n} \mathbb{R}^{2\ \ n}$$

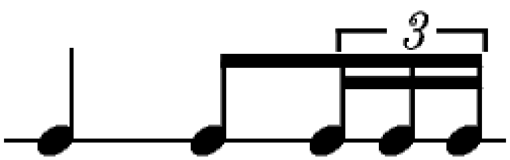


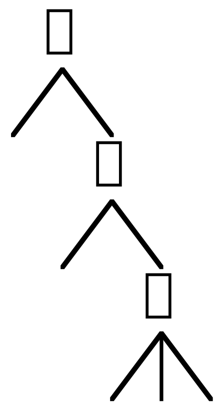
even subdivision of an interval into n subintervals

$$t,d \longmapsto t,d' \ ,\ \ t+d',d' \ ,\ldots, \ \ t+\ n\ 1\ d',d'$$

where $d'=d/n$

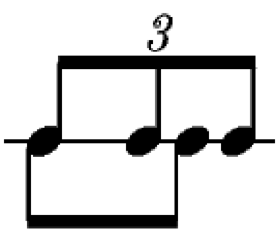
Application: *Rhythm Trees*





$\Xi(\alpha,(H,\Xi(\alpha,(H,\Xi(\alpha,(H,H,H))))))$

where H=Hum



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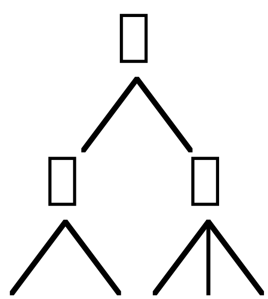


$\Xi(\Delta,(\Xi(\alpha,(H,H)),\Xi(\alpha,(H,H,H))))$

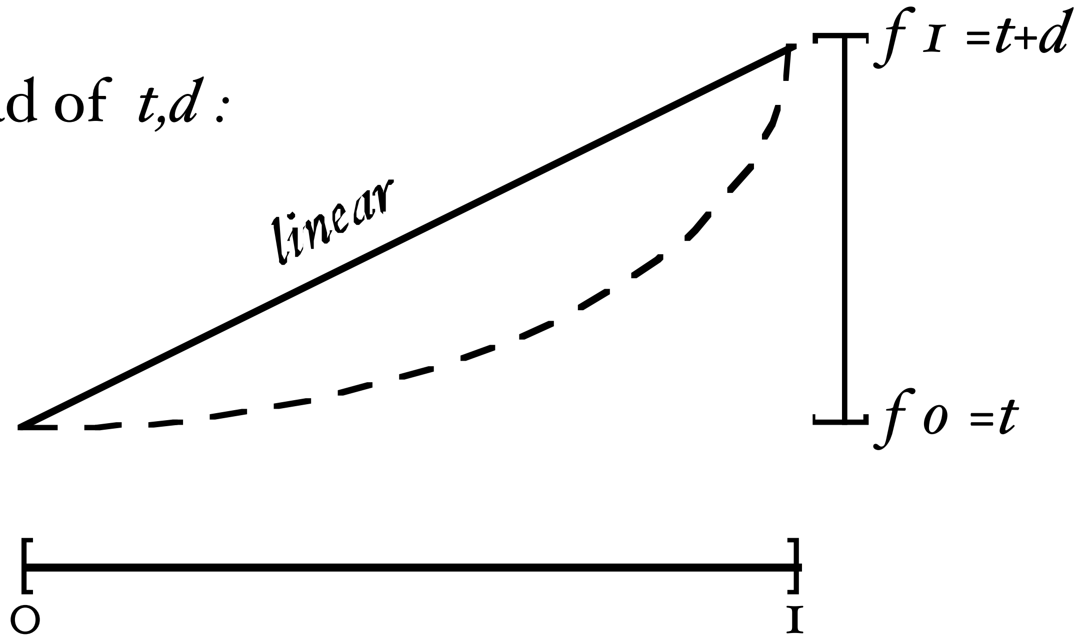
These functions, evaluated at *0,1* give the corresponding rhythms performed in the time interval *0,1* .

Instead of Time Vectors,

Functions of the Unit Interval

$$0,1 \to \mathbb{R}$$

Instead of t,d :

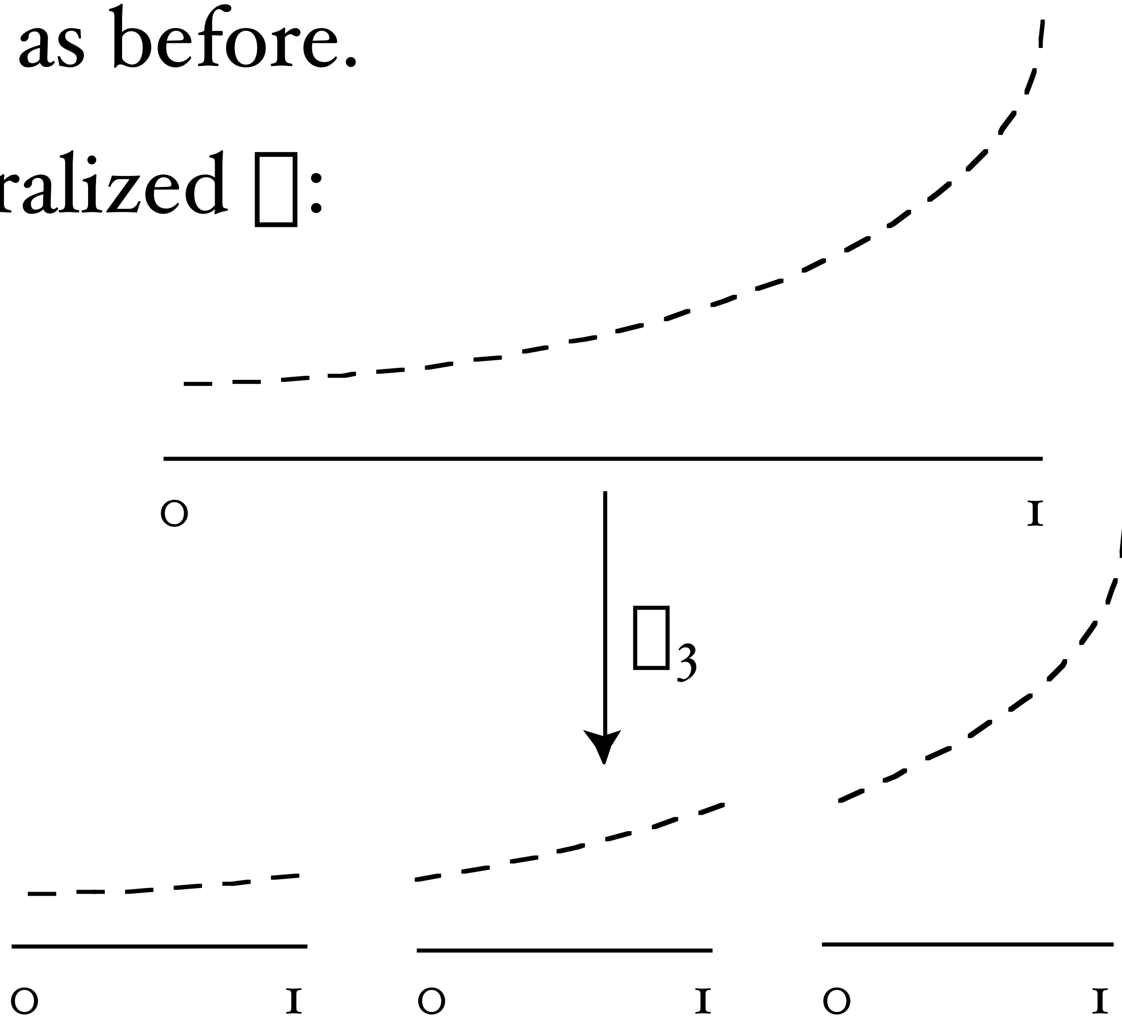


But, we can use nonlinear functions to achieve accel and deceleration and expressive rhythms!

Inheritance Functions for $0,1 \to \mathbb{R}$

□ just as before.

Generalized □:



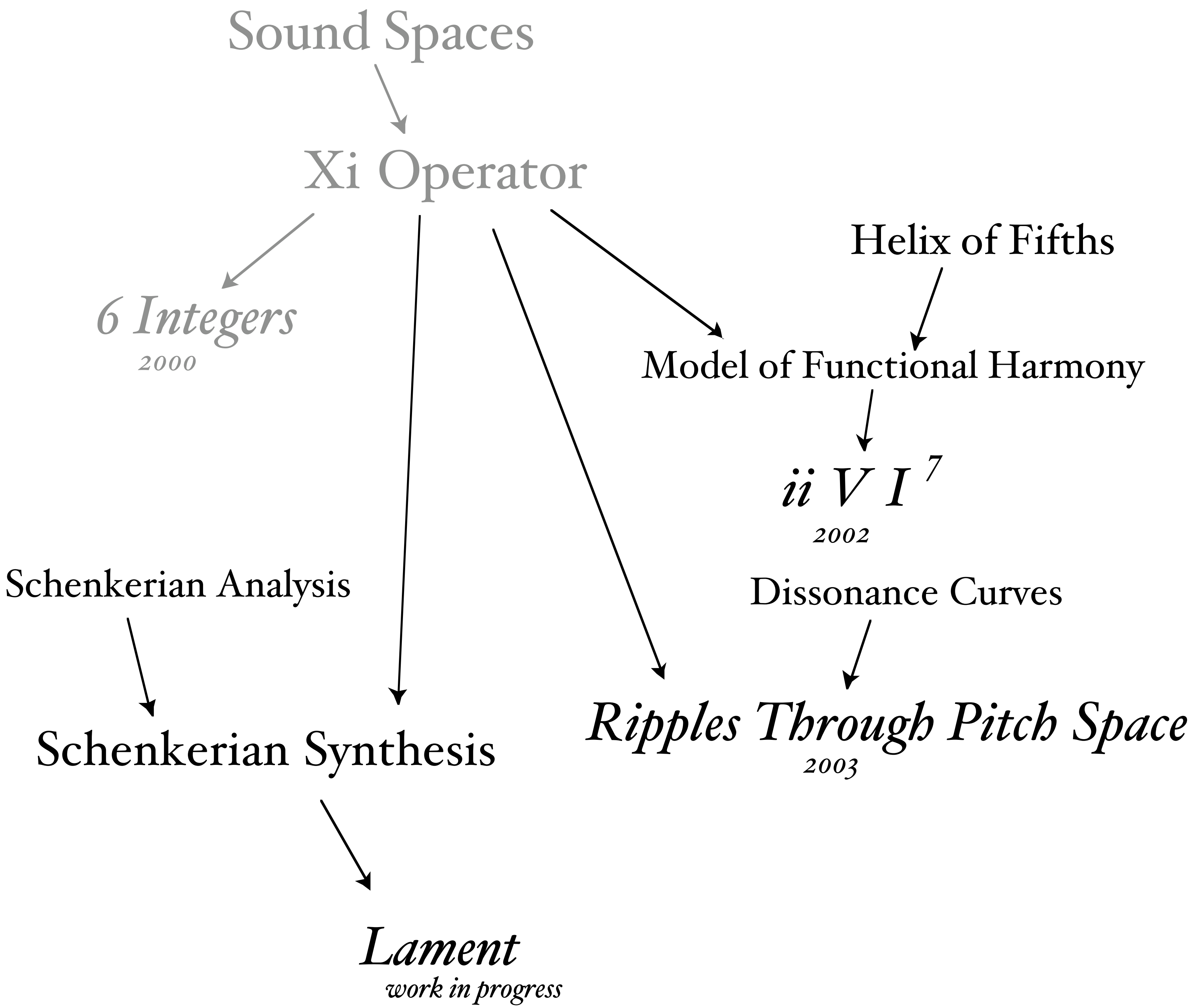
But why not just use time vectors and apply a global time map at the end?

The hierarchical $0,1 \to \mathbb{R}$ approach permits local modification of the time map. Furthermore, different simultaneous components of a piece can have distinct time maps!

Products of Inheritance Functions

We can form products of inheritance functions and thus pass several attributes of sound through the tree at once in parallel.

E.g. rhythm, pitch, harmony, dynamics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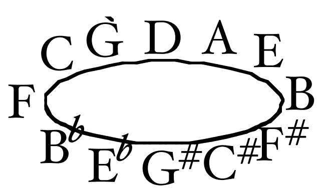
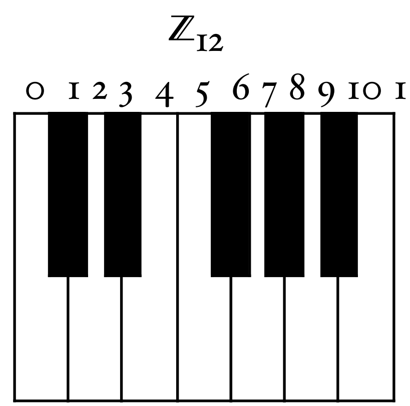
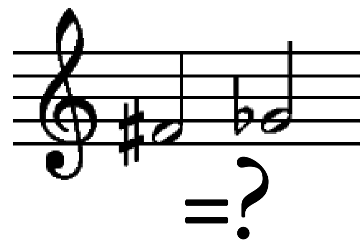
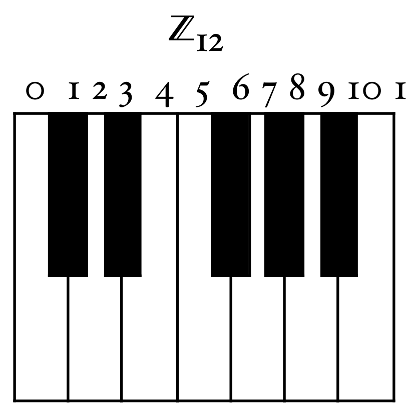


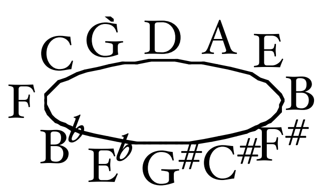
"Circle of Fifths"



“Enharmonic equivalence”:







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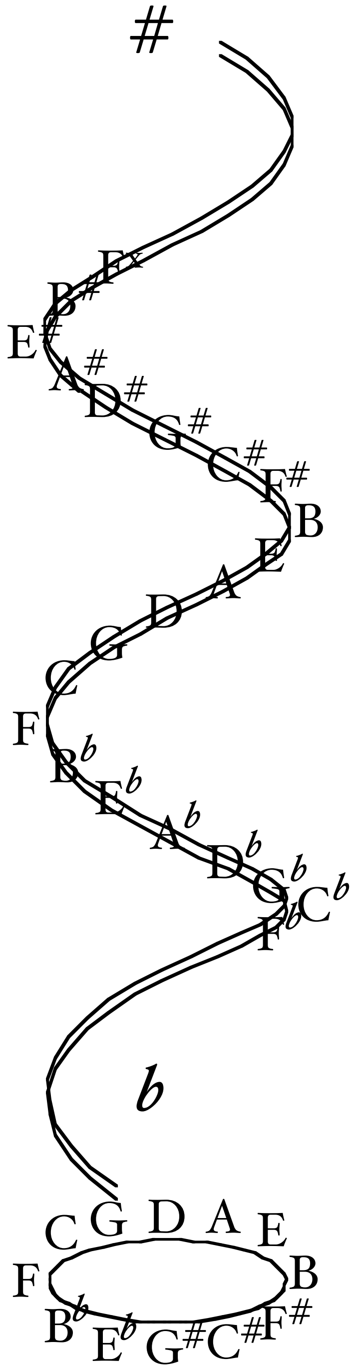

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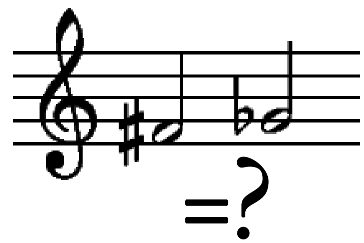
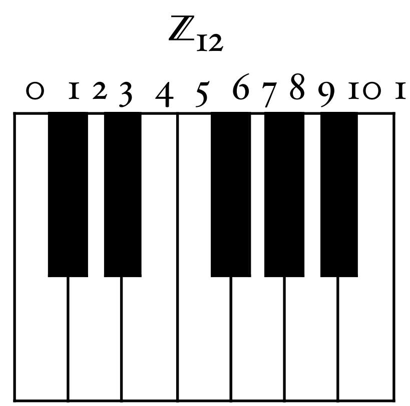
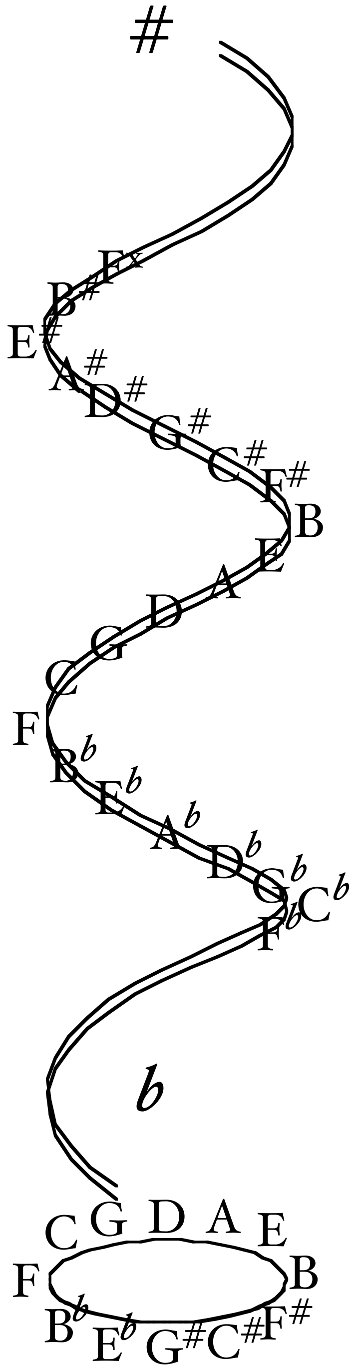




Helix of Fifths



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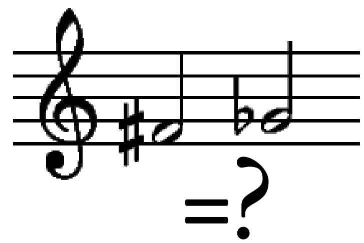
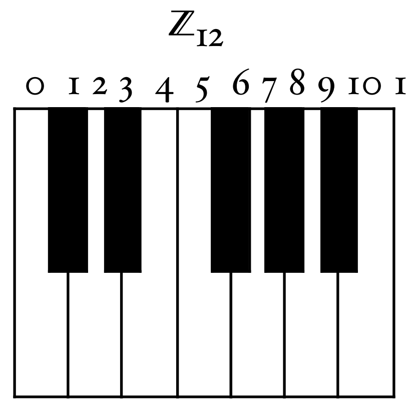
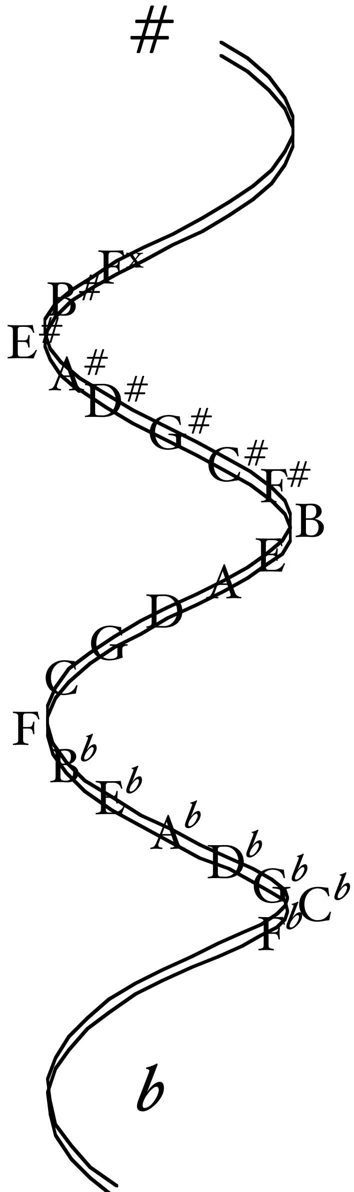




Helix of Fifths

Strongly inspired by the work of Eric Regener



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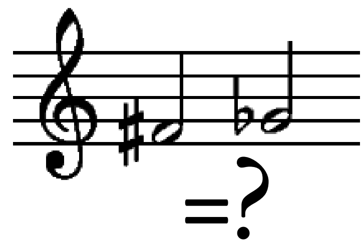

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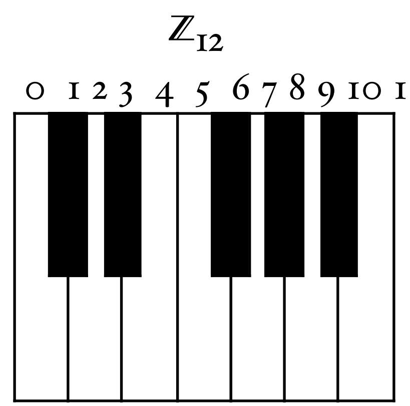
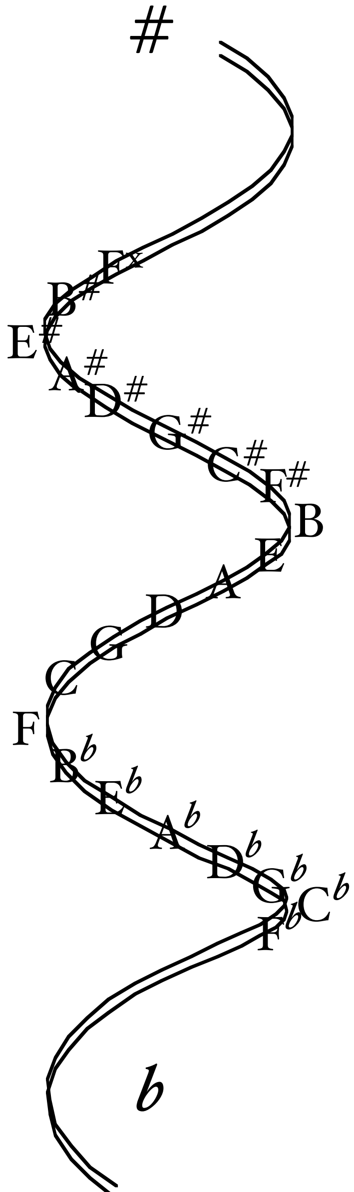




Helix of Fifths

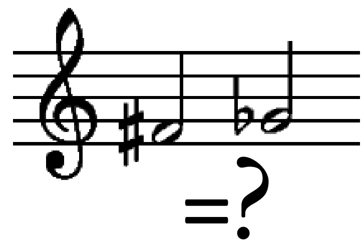
Strongly inspired by the work of Eric Regener

Give it the algebraic structure Z,+ .

$$nota(n) := \left(n \bmod 7, \lfloor n/7 \rfloor\right)$$

letter name, accidental

“Enharmonic equivalence”:



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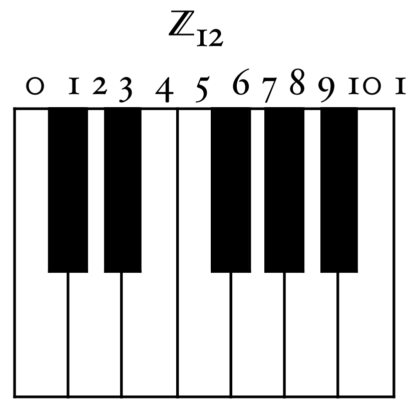

Then, look at the sublattice:

$$\overline{H} \coloneqq \left\{ (h,p) \square Z^2 : 4h \square p \square 7Z \right\} \square (Z^2,+)$$

where *h* is helix position and *p* is staff position.

It has a positive cone:

$$P \coloneqq \left\{ (h,p) \square \overline{H} : p \geq 0 \right\}$$

and a corresponding absolute value:

$$|(h,p)| \coloneqq \left(h \cdot sign(p), |p| \right).$$

Brief Introduction to Functional Harmony

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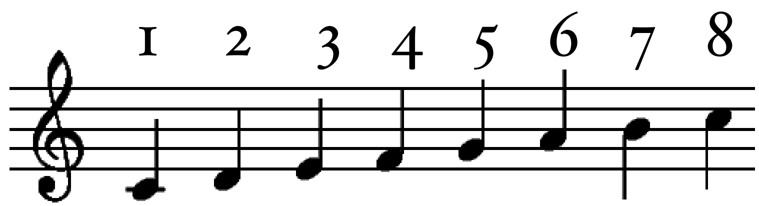

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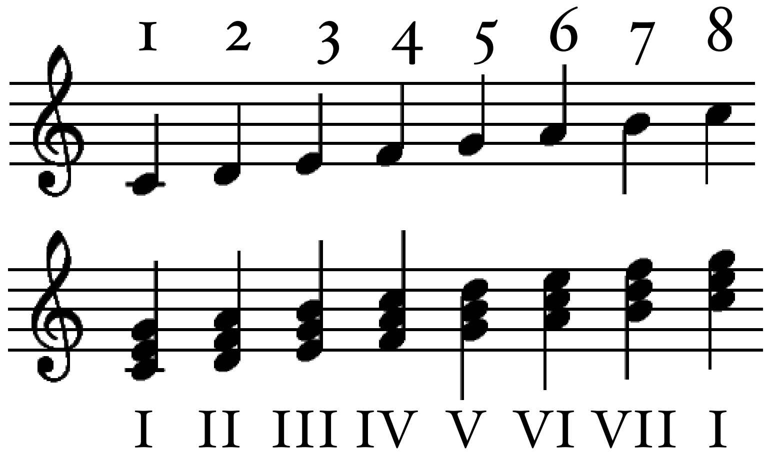
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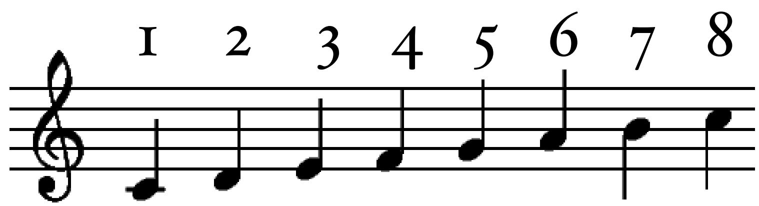

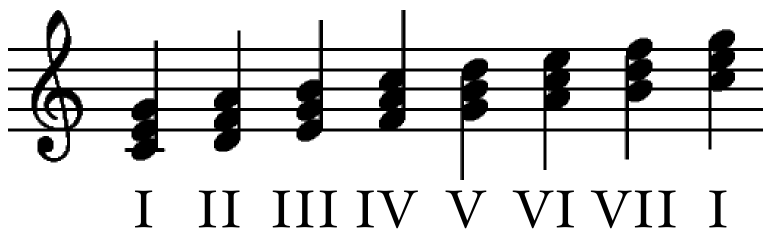

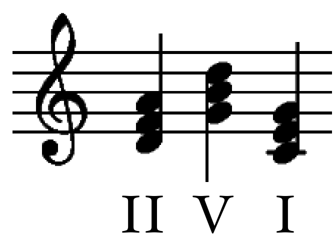

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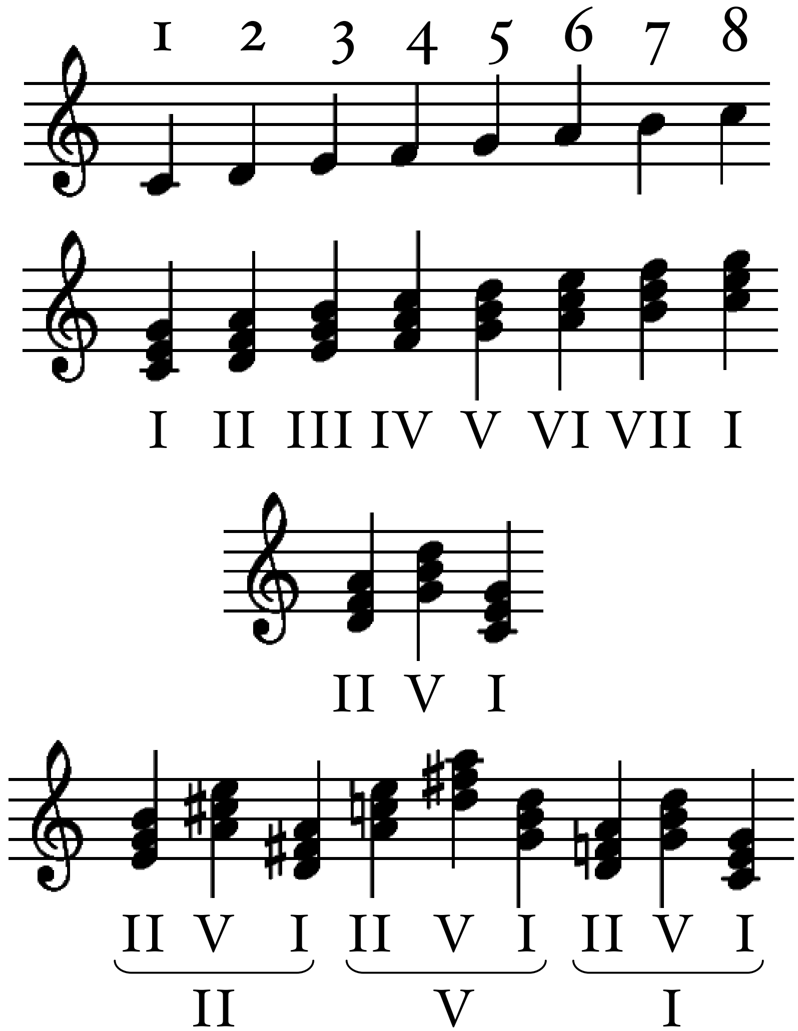
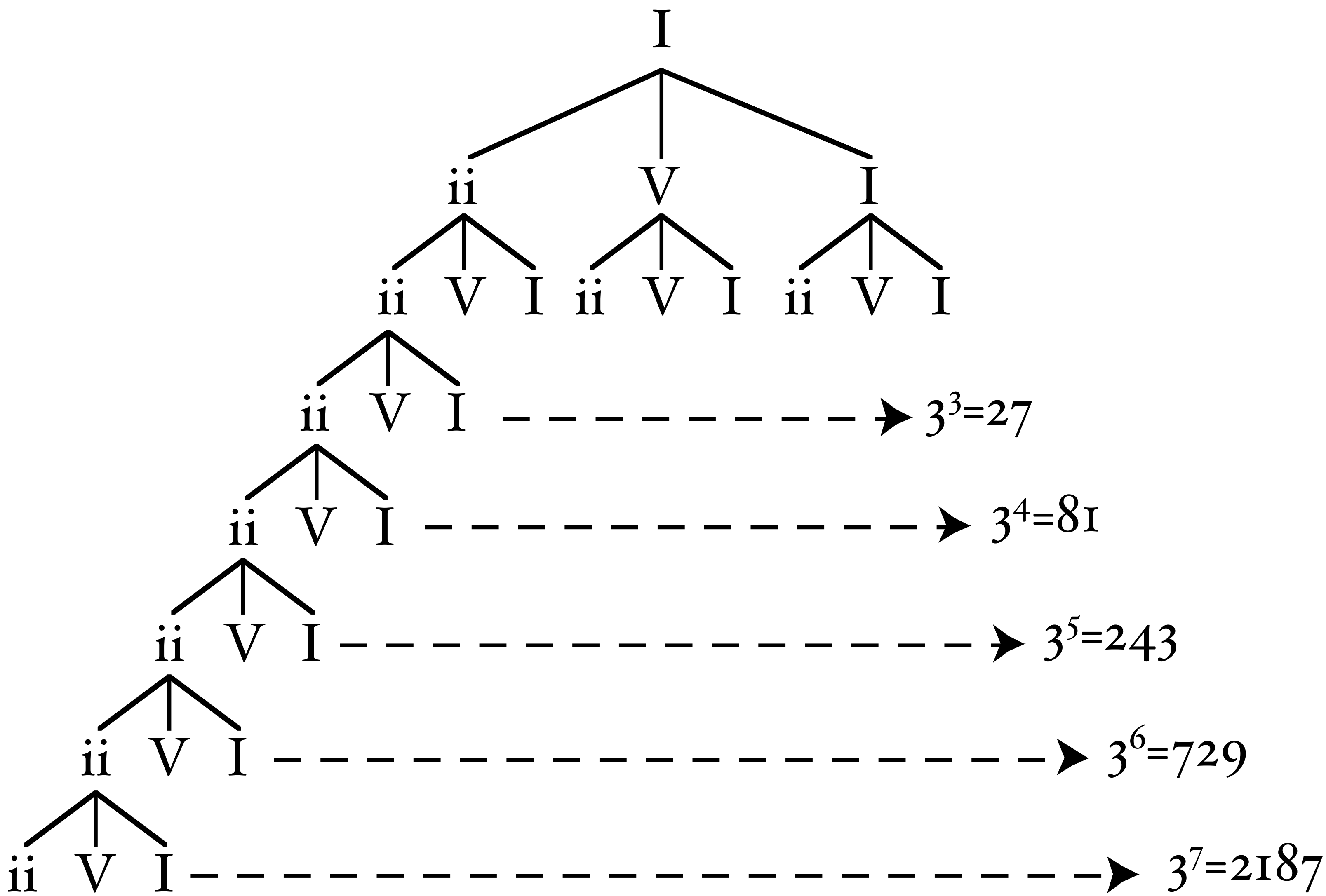
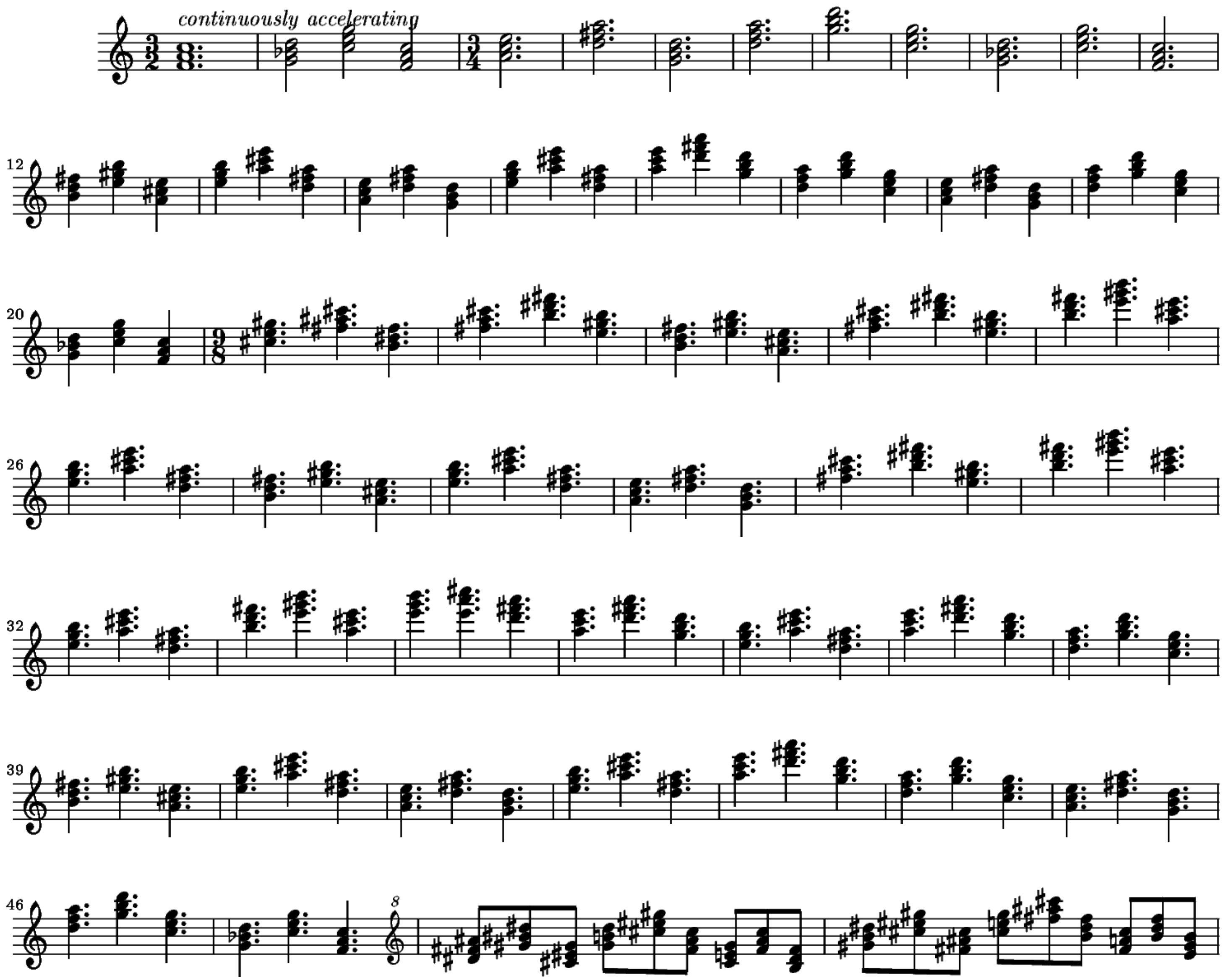
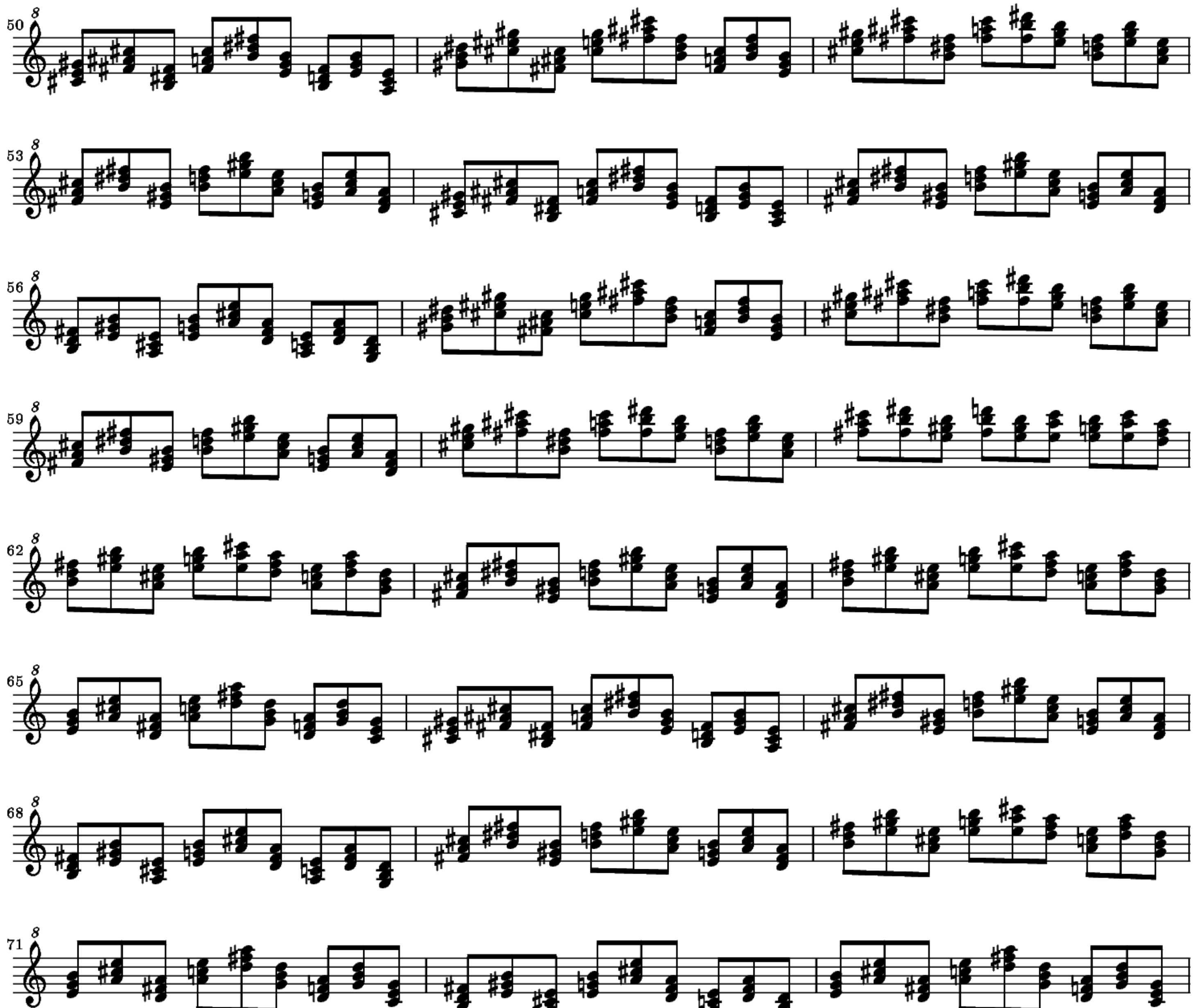


ii V I ^{7} *2002*



total progession length: 3280

(ii–V–I)^7

Carl McTague


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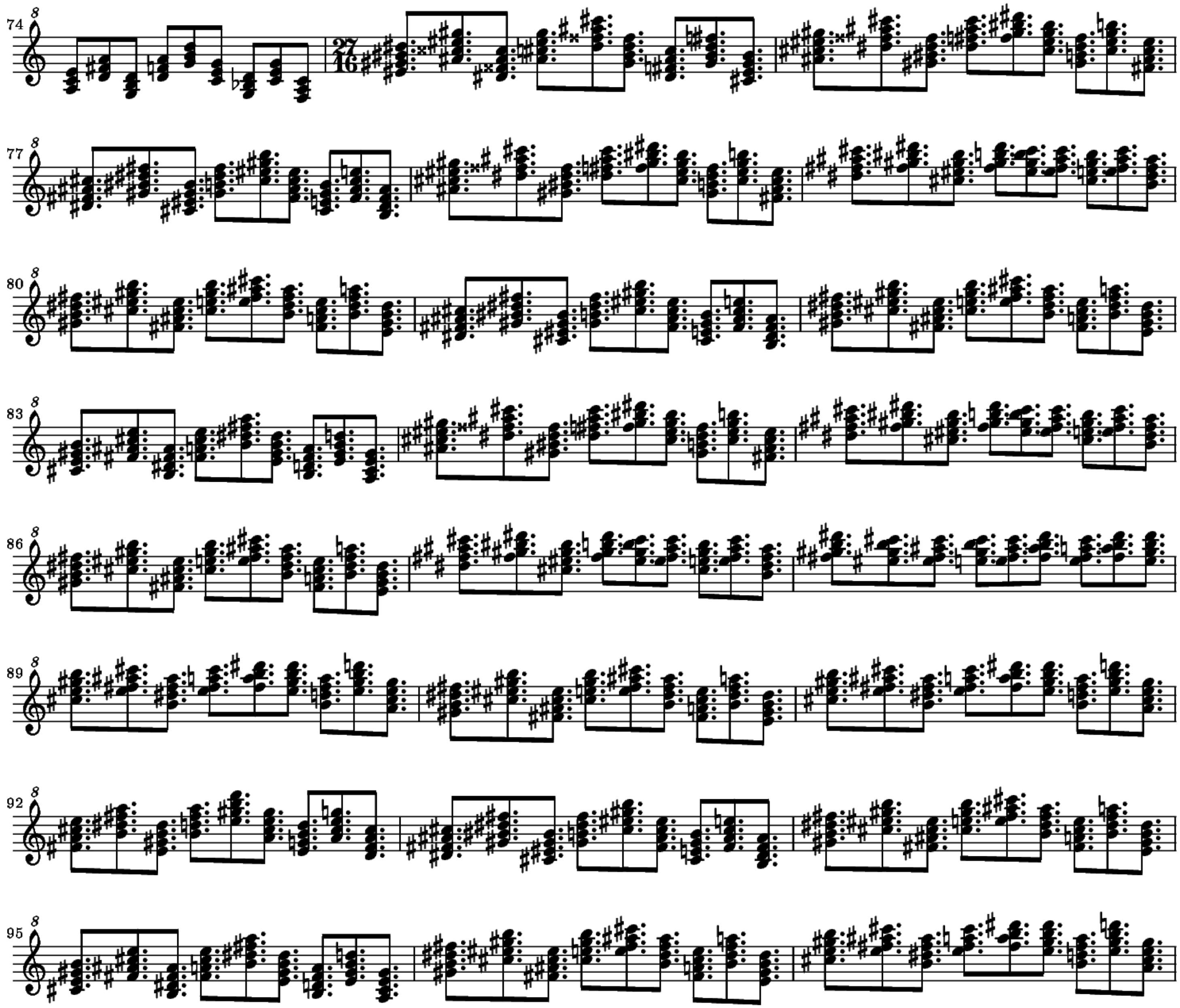

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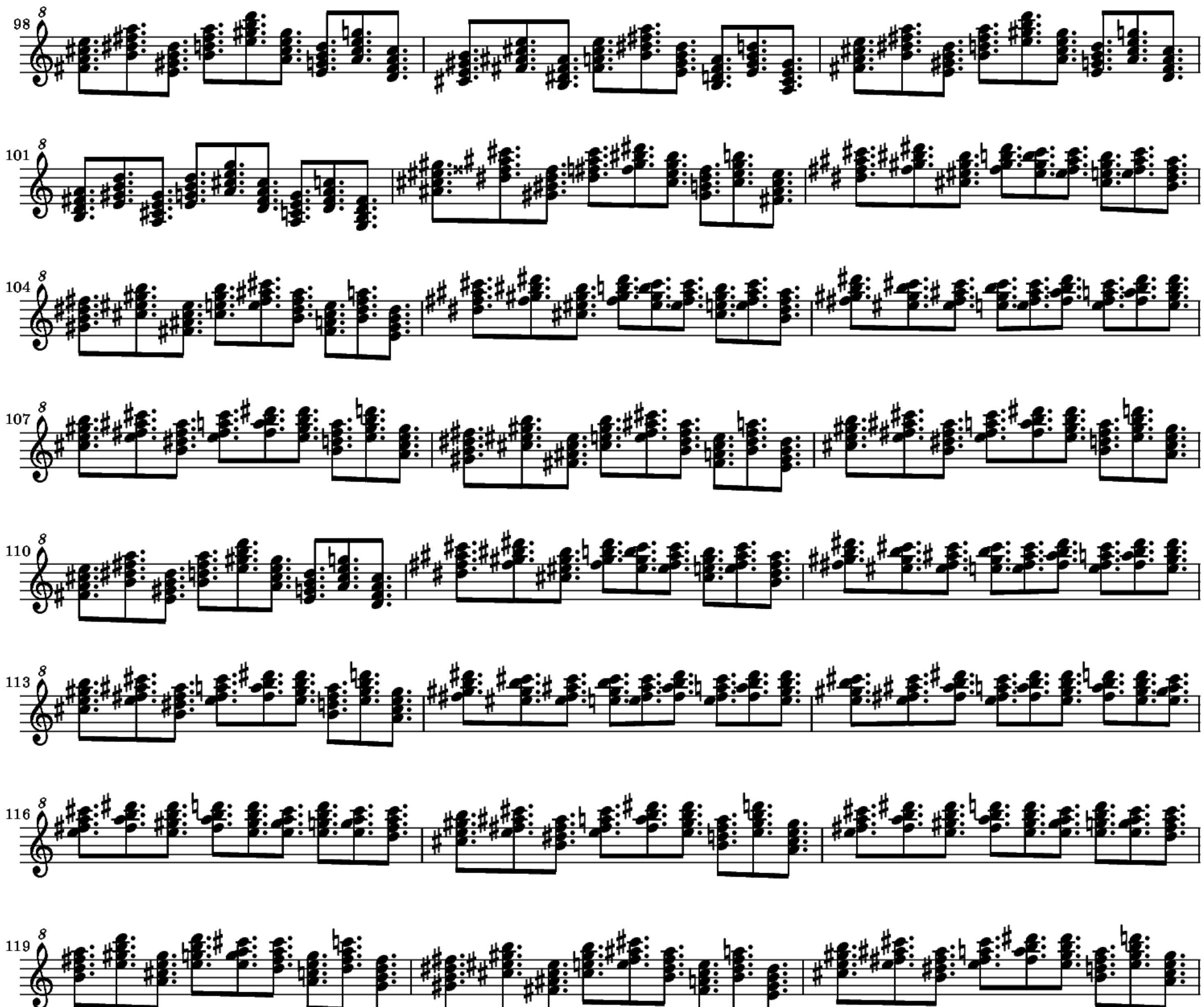

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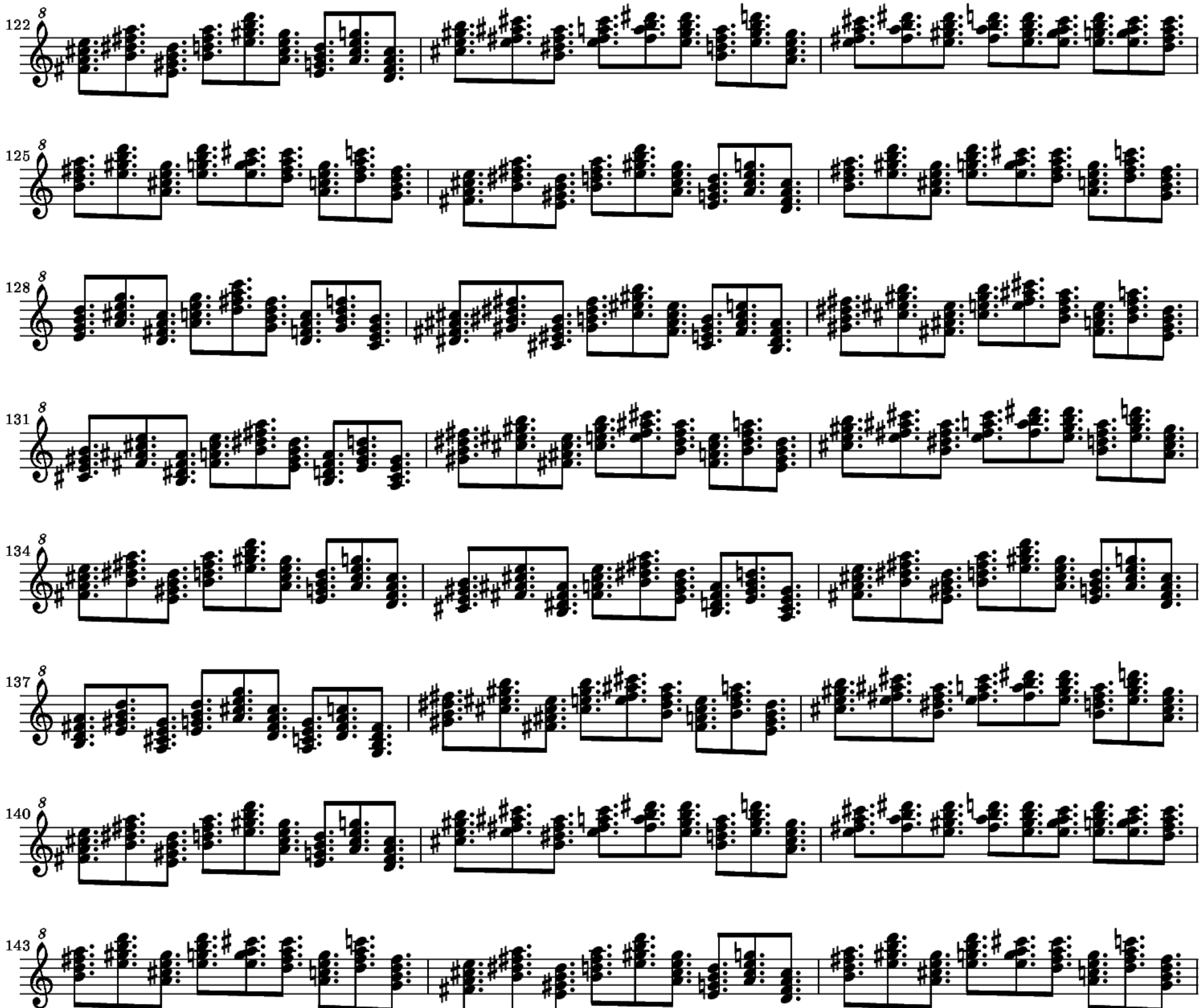

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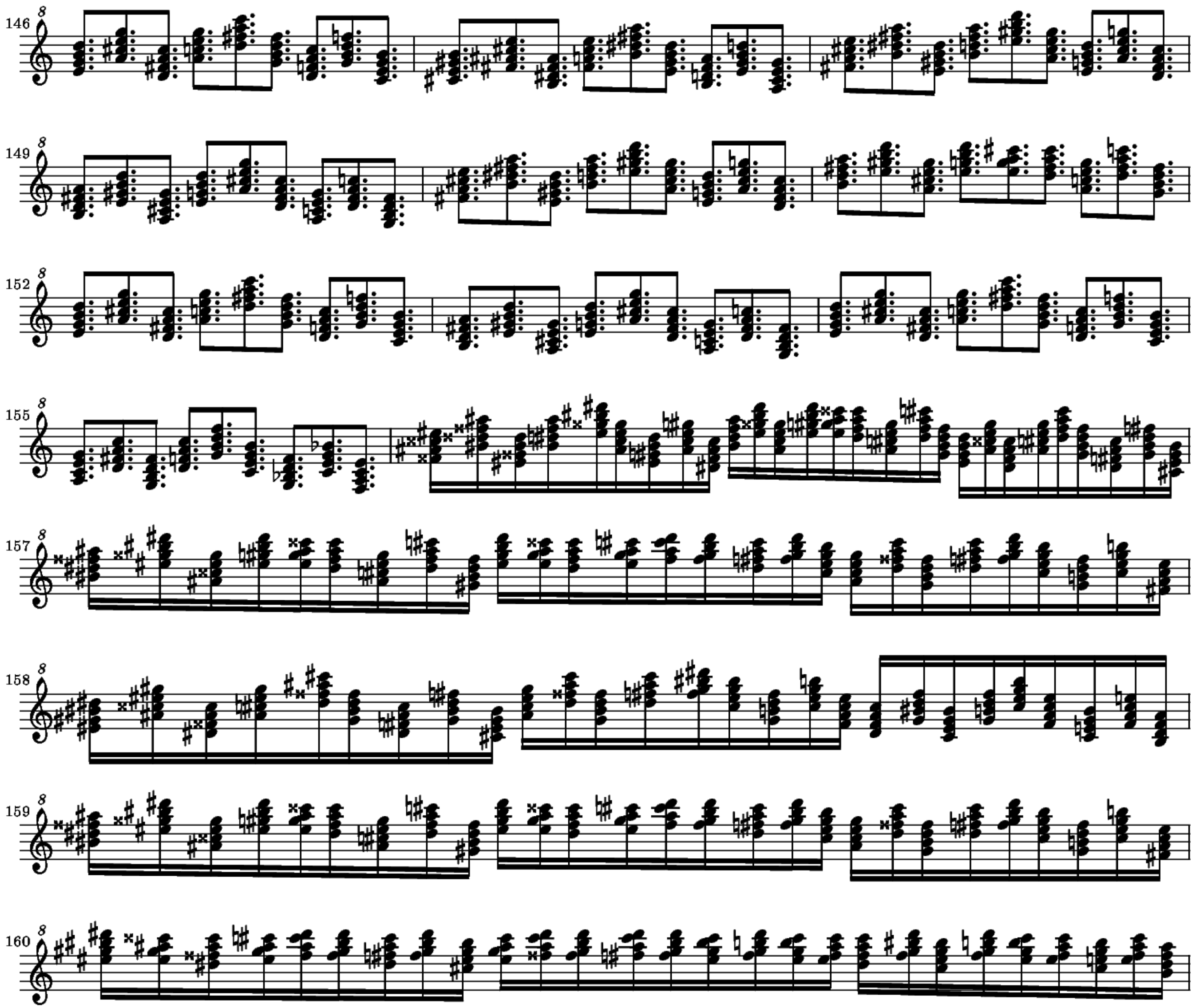

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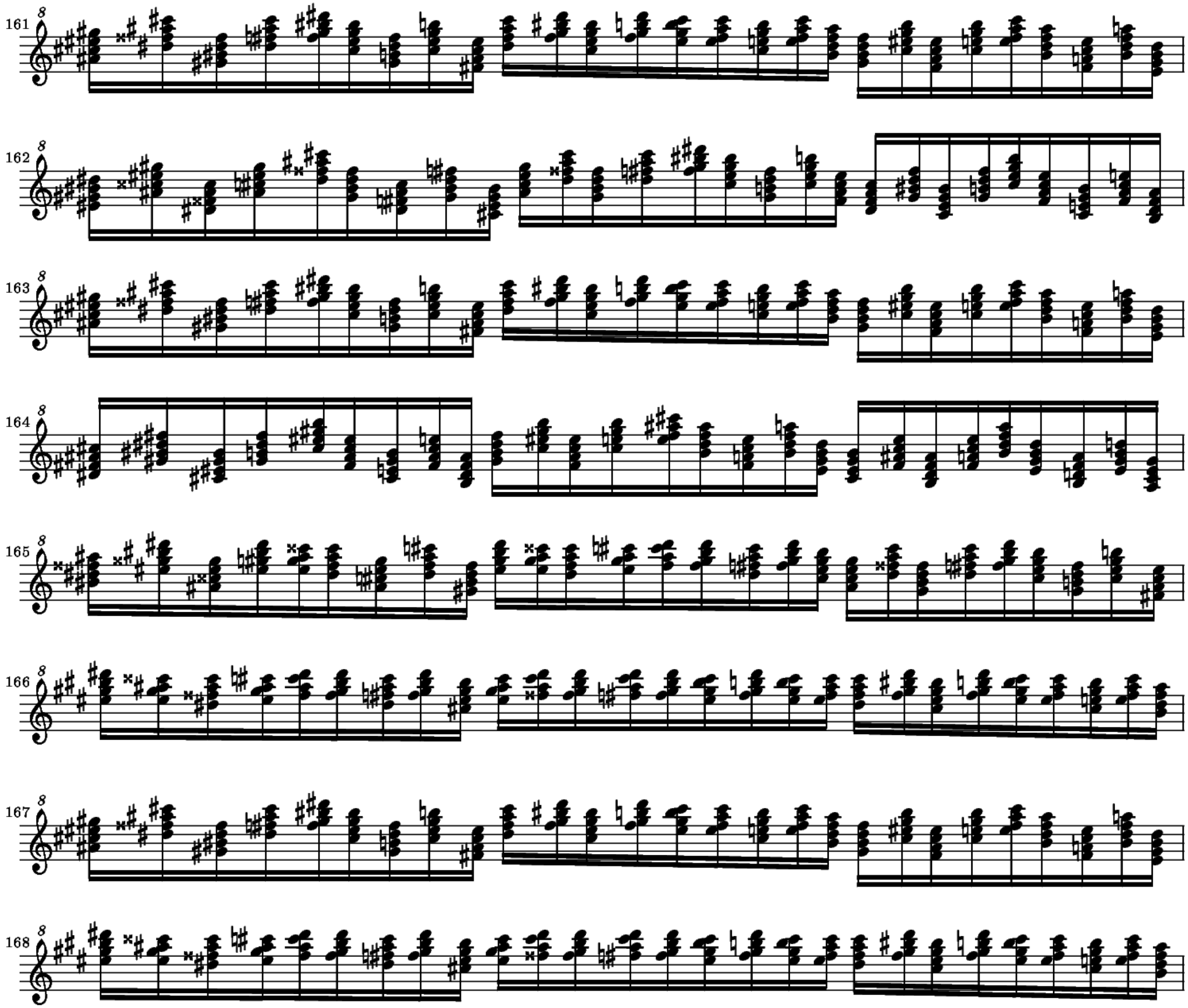

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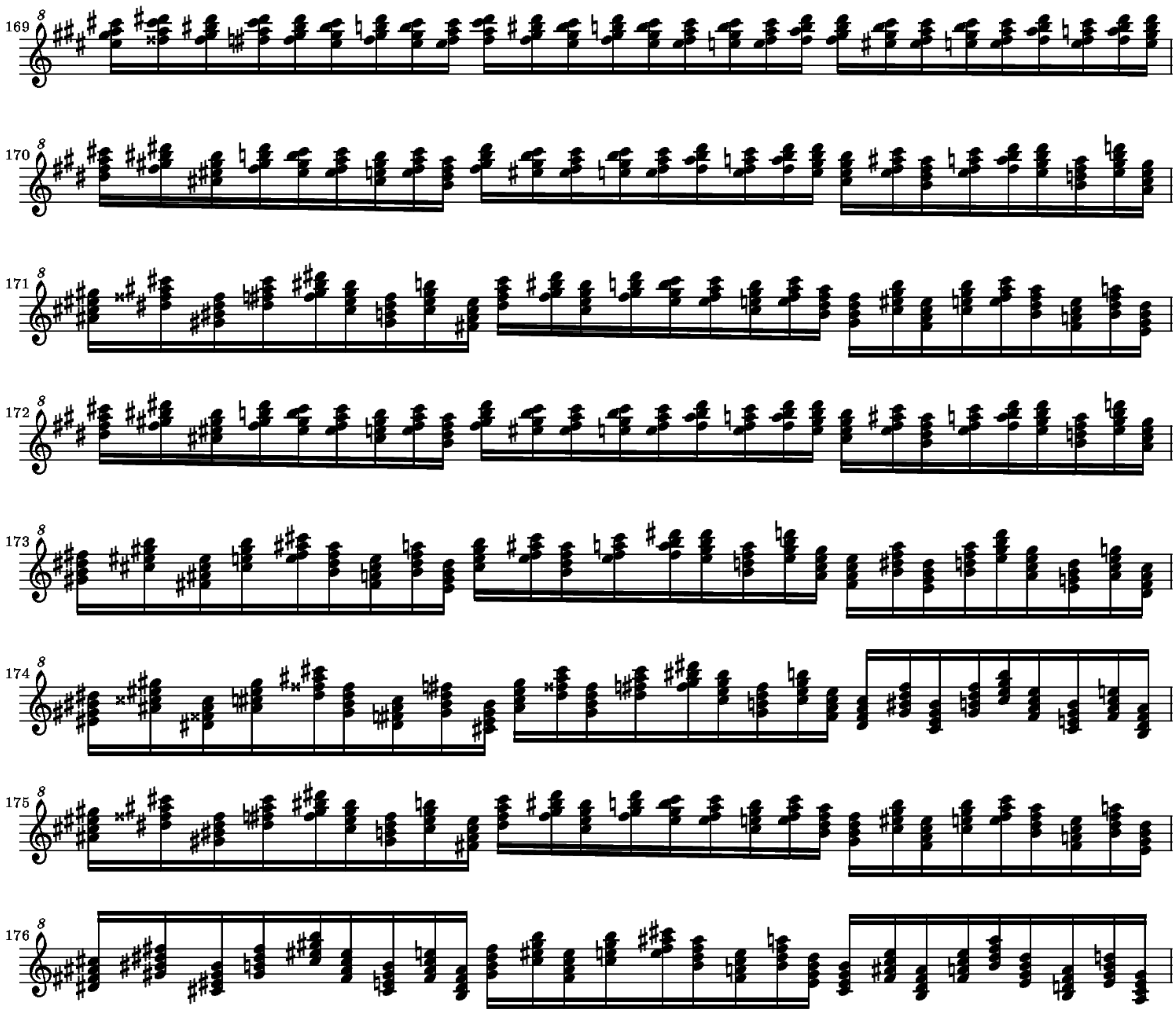

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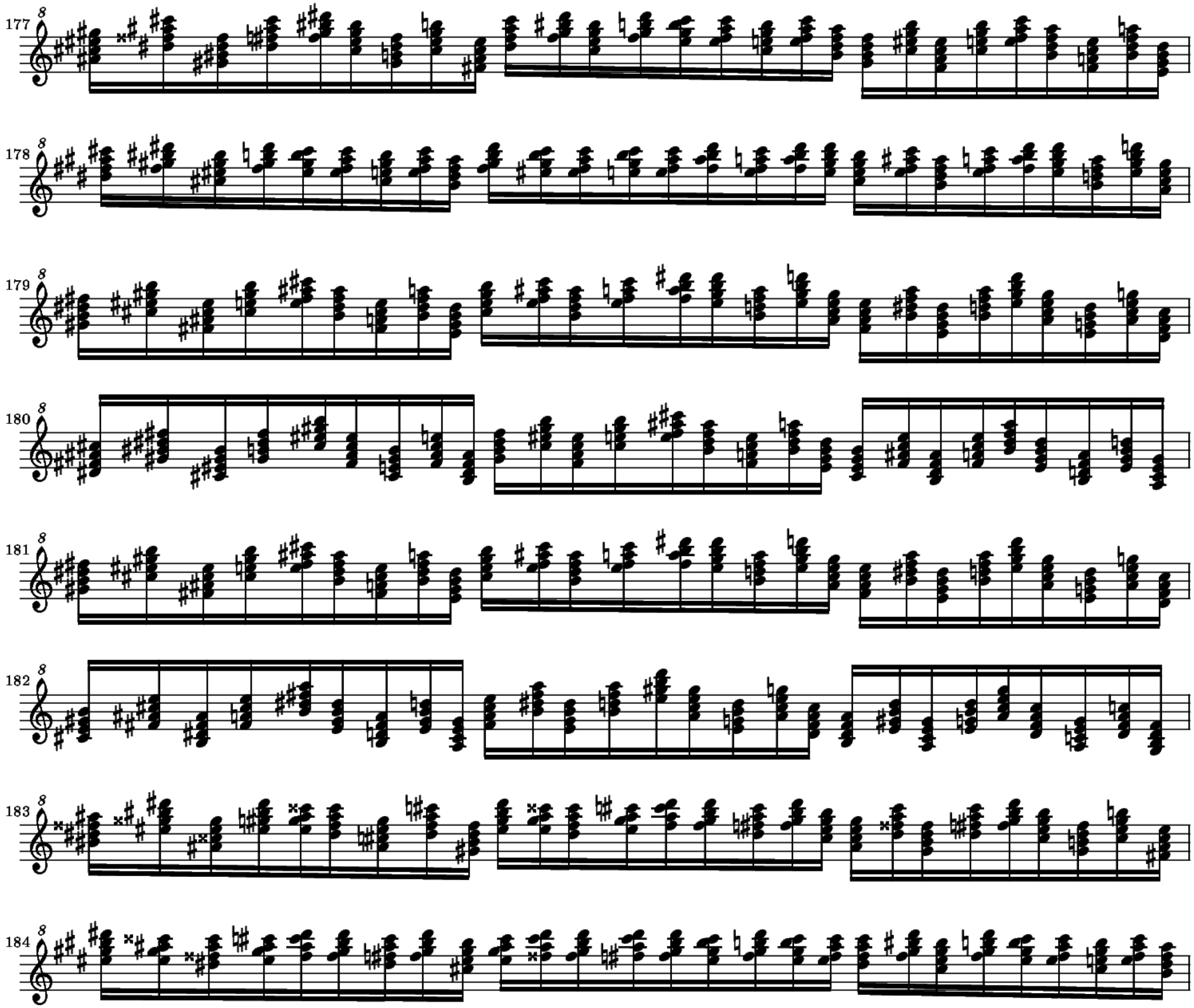

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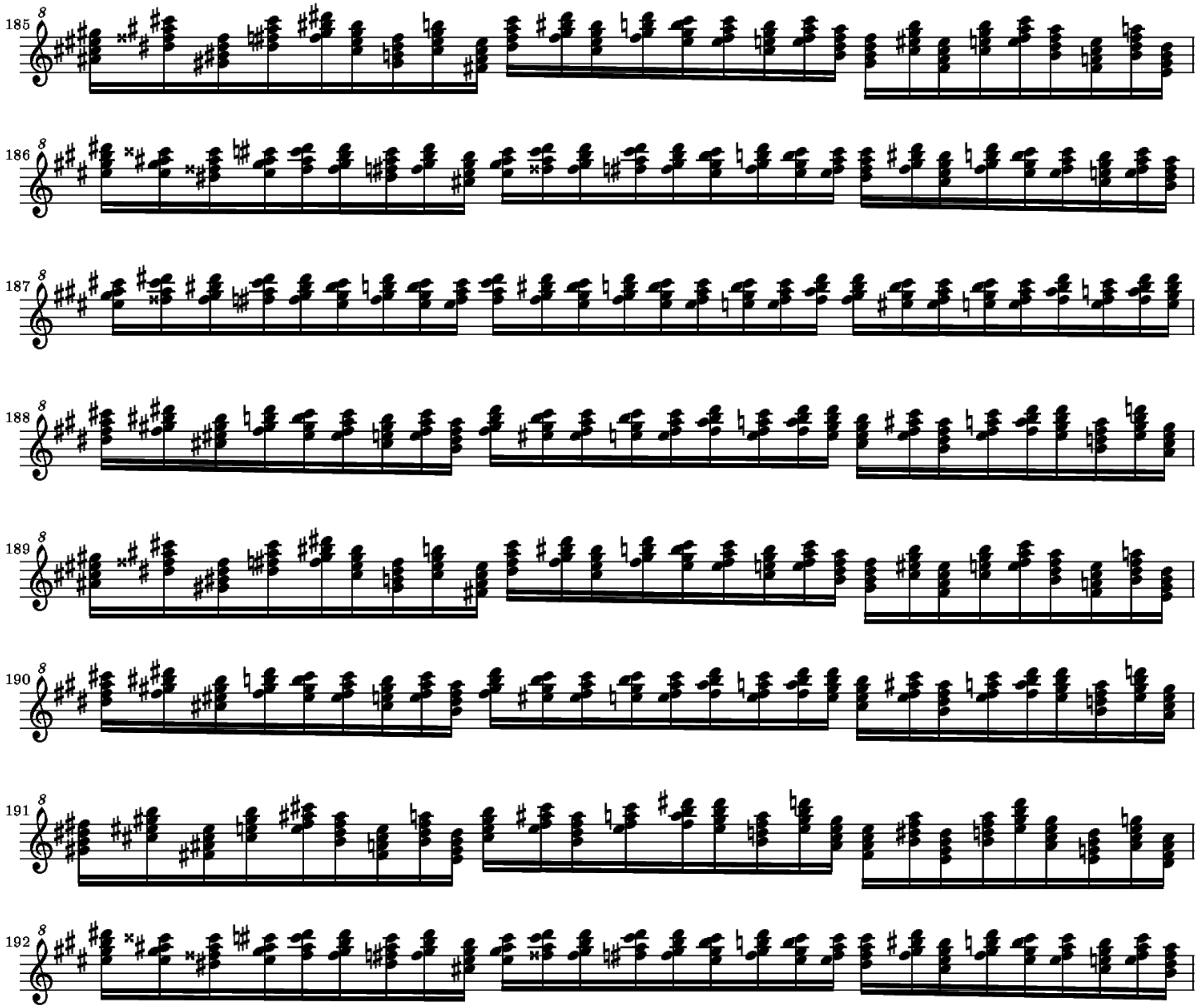

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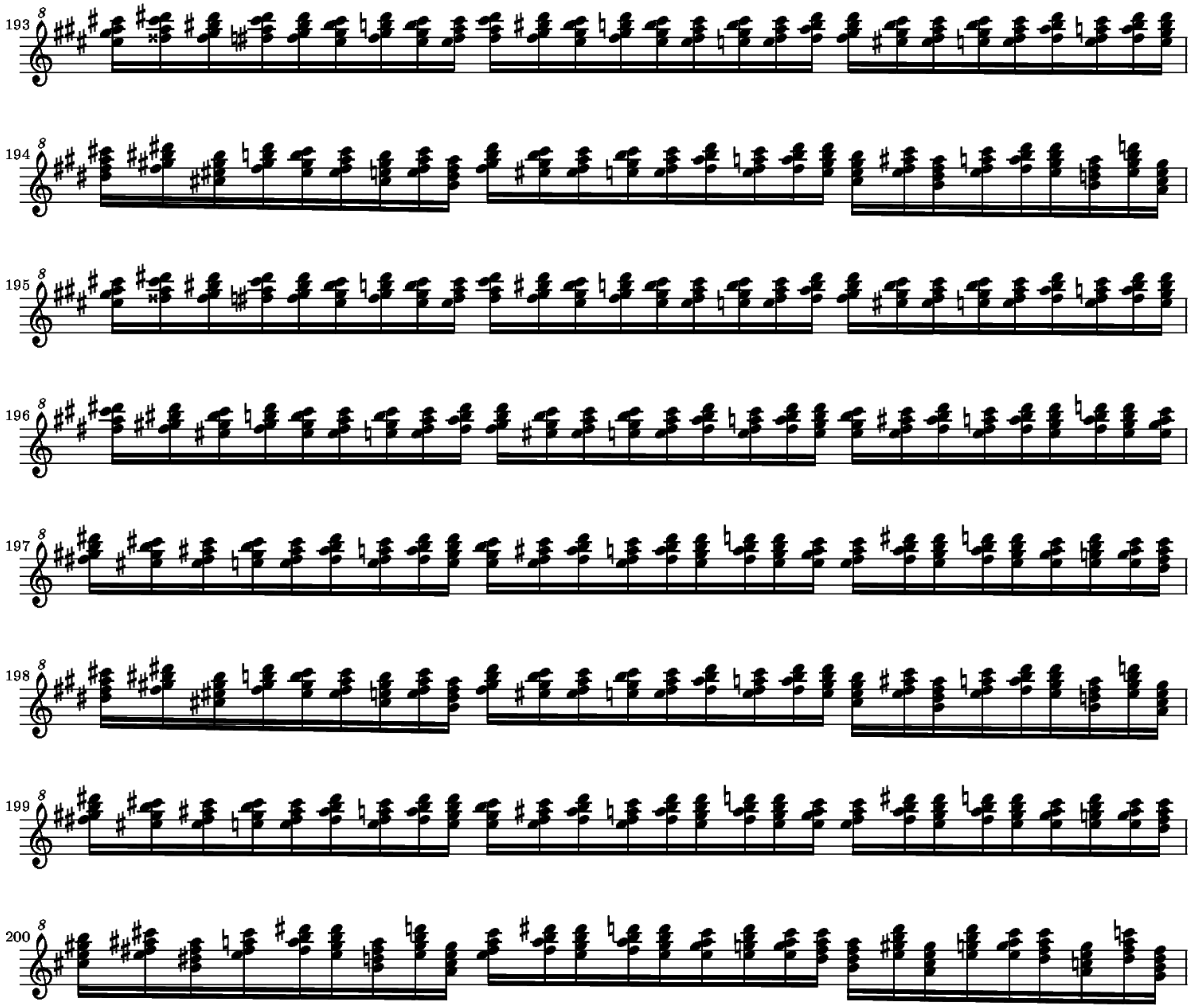

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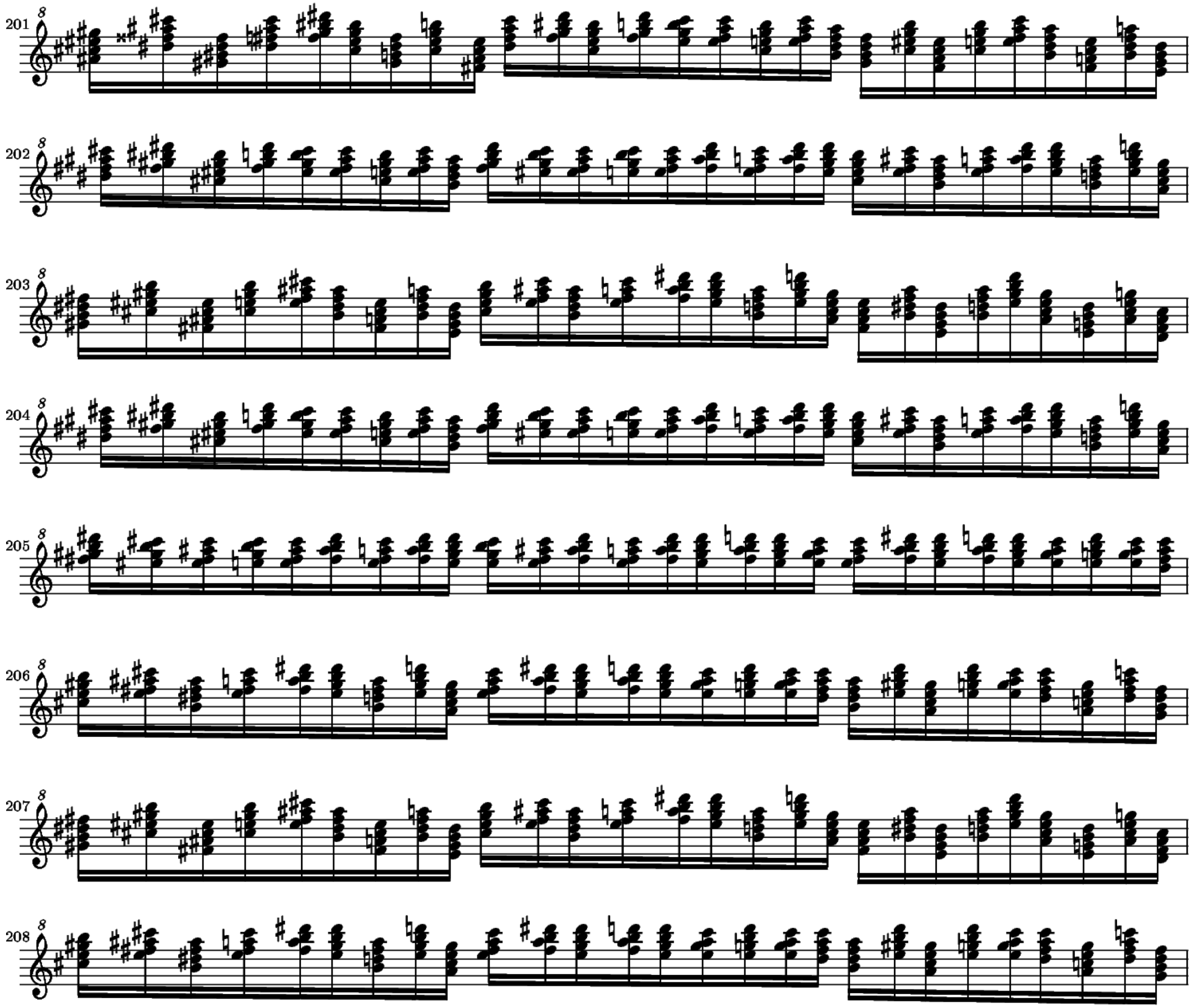
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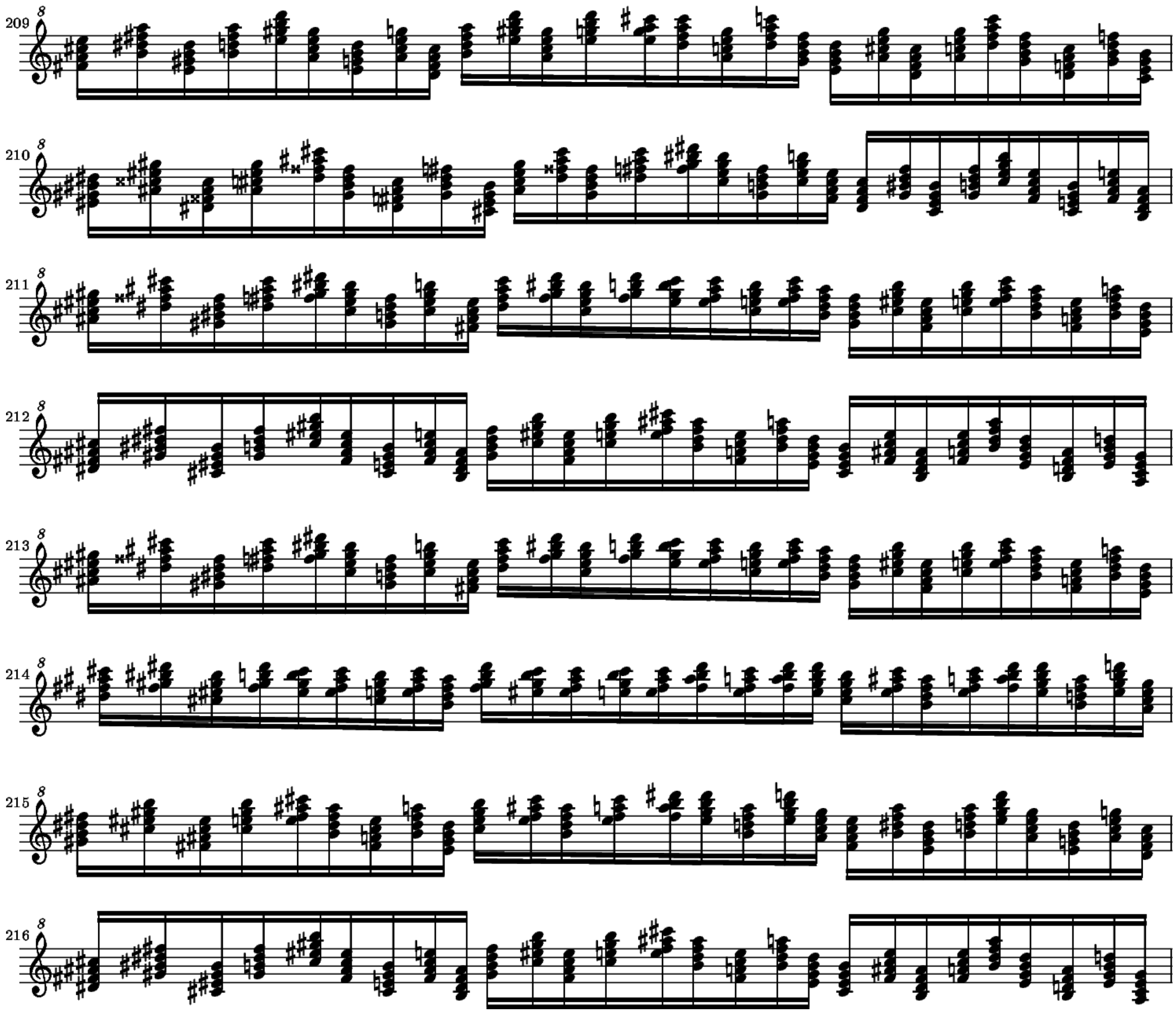

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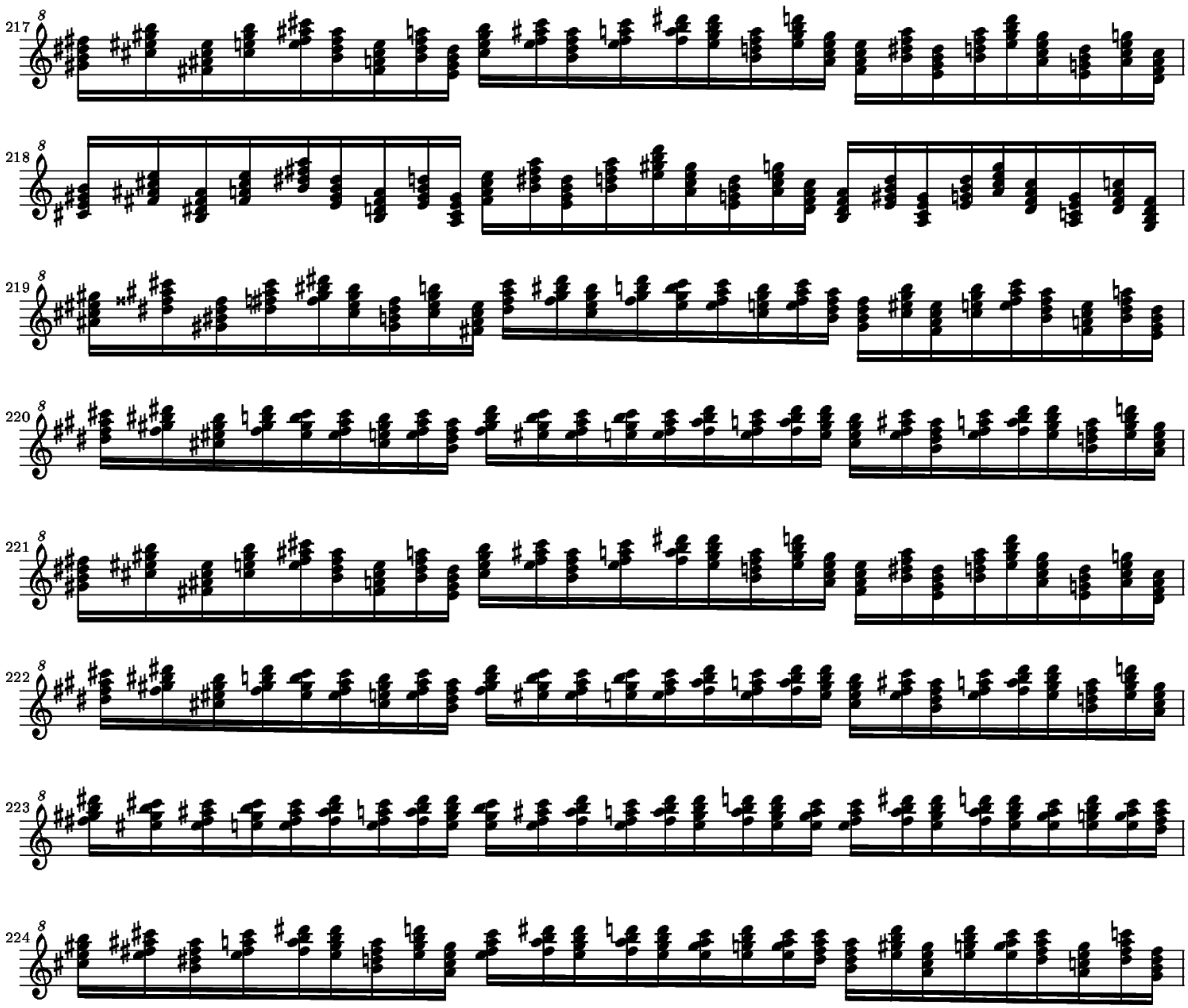

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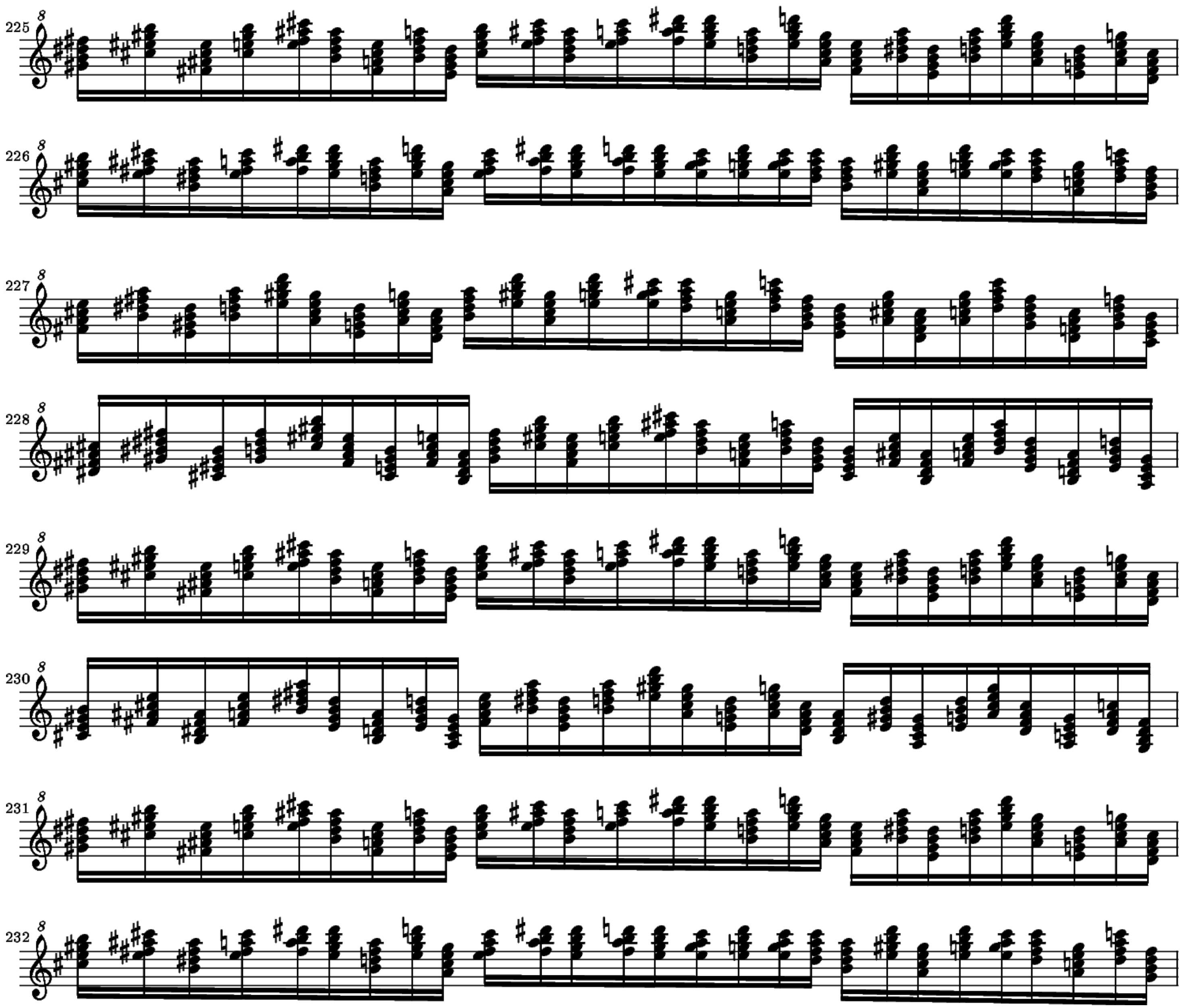

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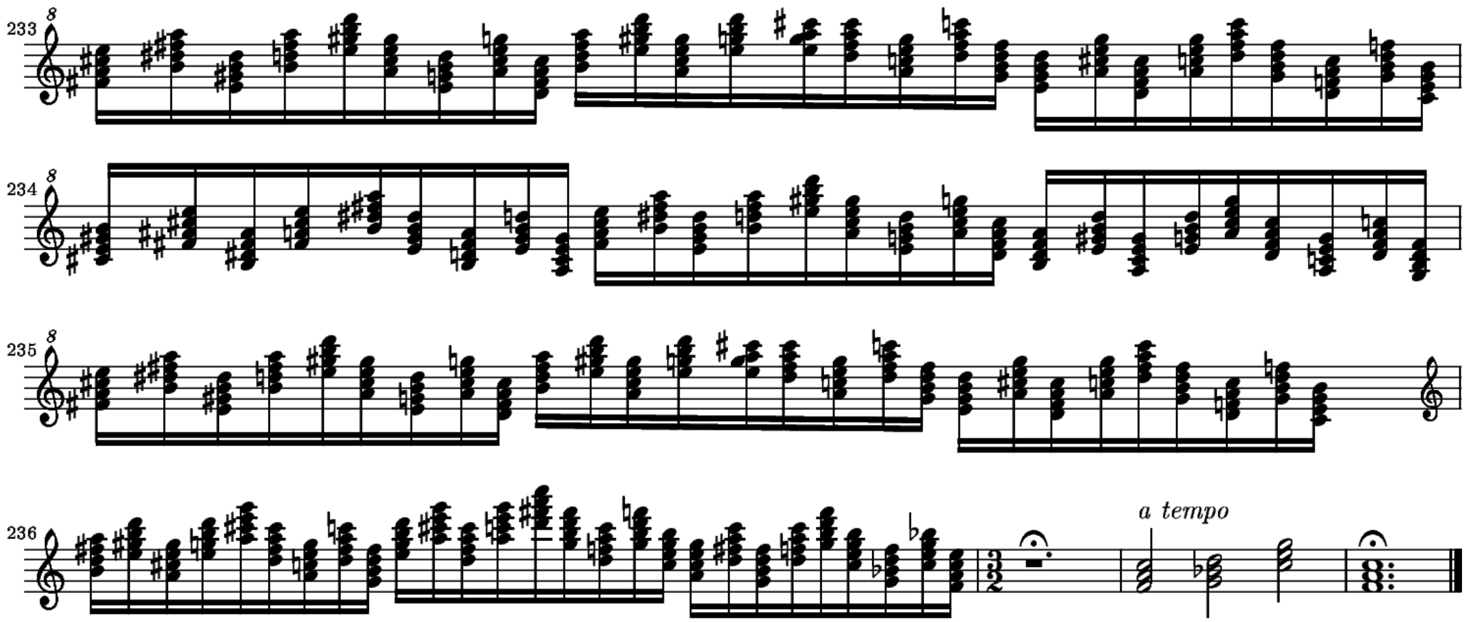

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8
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a tempo

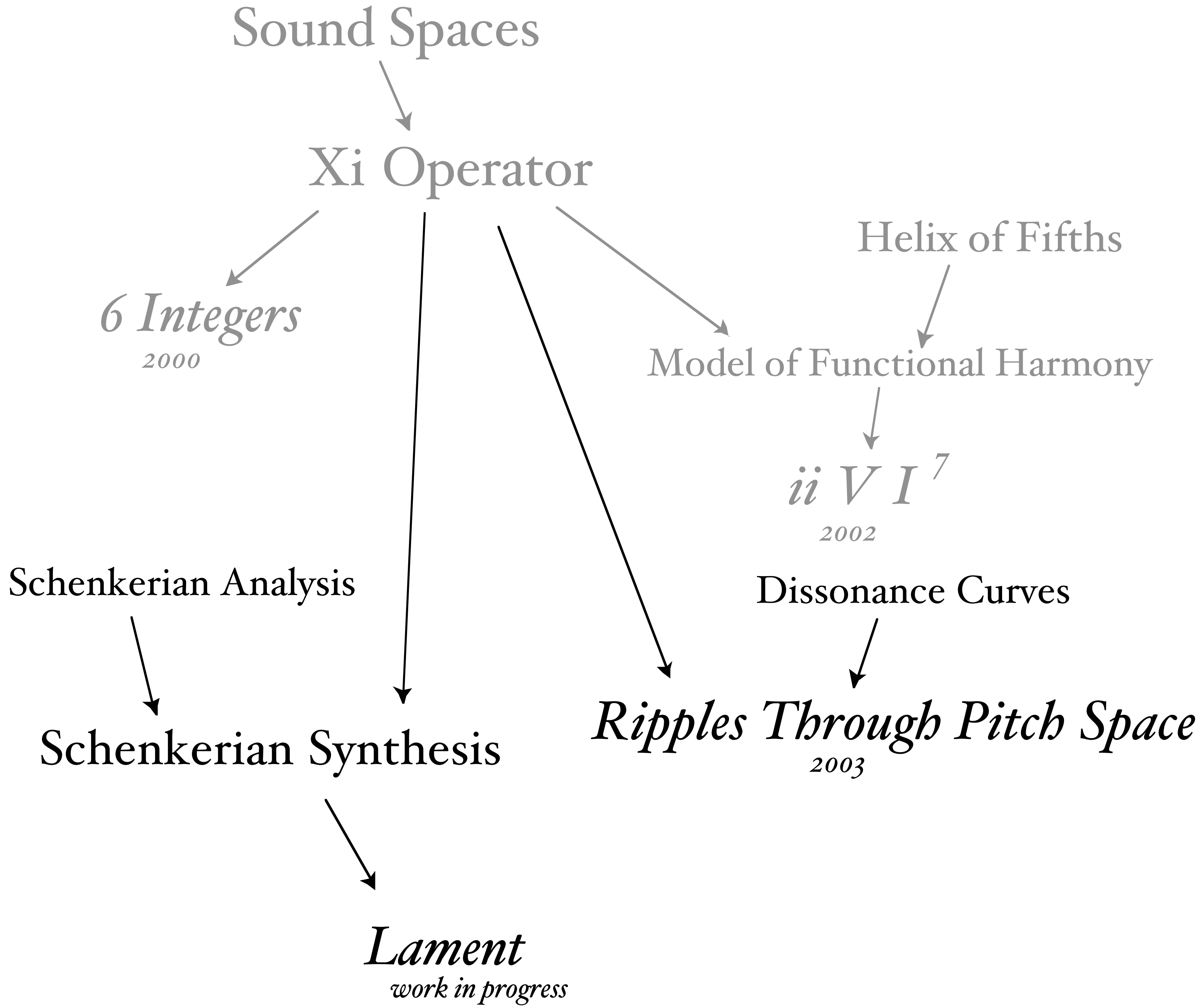
Here, the helix was fitted with
"3 limit tuning."

More generally: use factorization of
rationals to biject $\mathbb{Q}$ into $\mathbb{Z}^\infty \hookrightarrow \mathbb{R}^\infty$:
finite support

$$\langle a_n \rangle \mapsto \prod p_i^{a_i}$$

p_i the i th prime

and lift a nice metric from $\mathbb{R}^\infty$.



Sound Spaces
Xi Operator
6 Integers
2000
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Model of Functional Harmony
ii V I 7
2002
Schenkerian Analysis
Dissonance Curves
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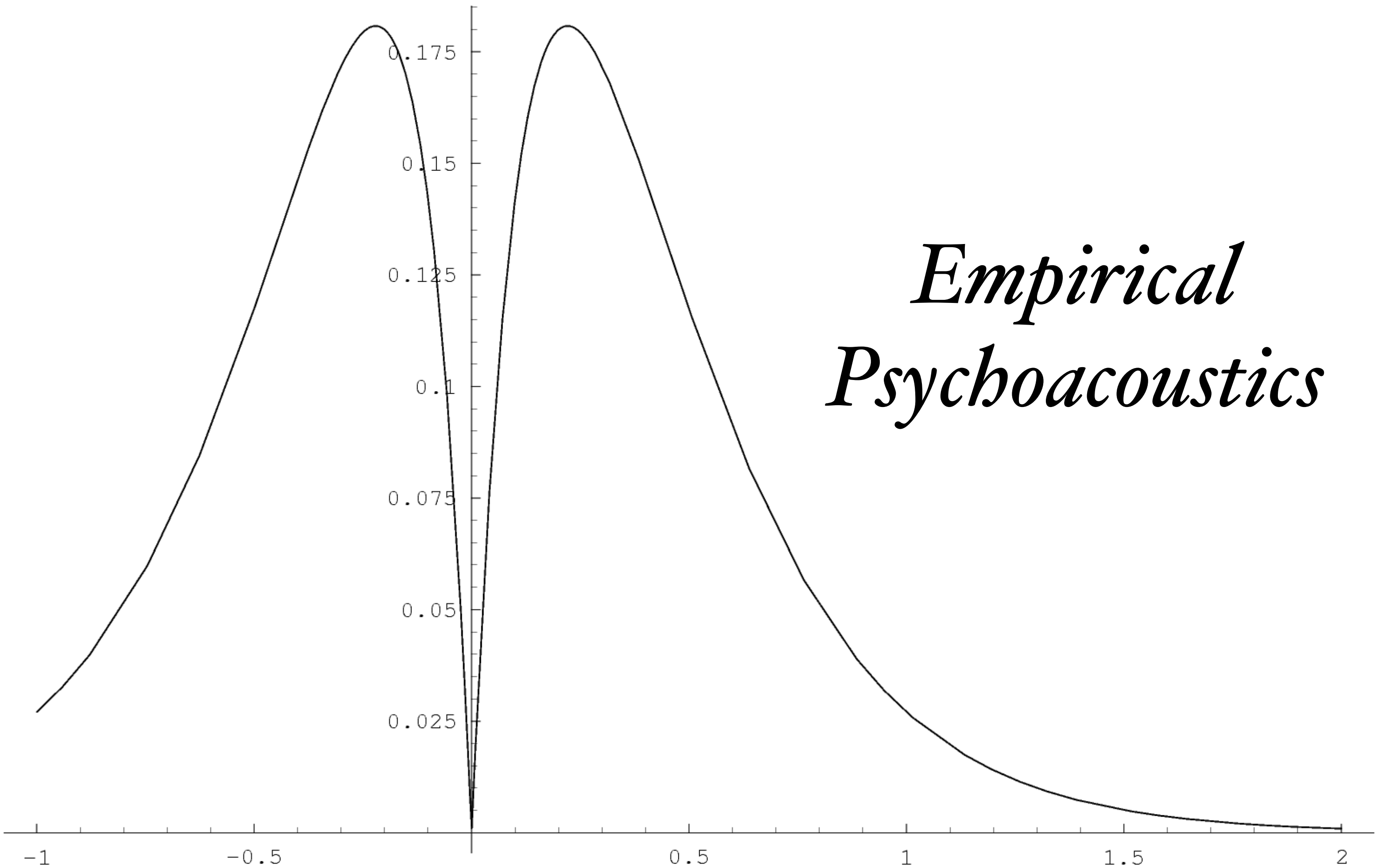
Ripples Through Pitch Space
2003
Lament
work in progress

Dissonance of 2 *Pure Sine Tones*

Kameoka, Kuriyagawa & Sethares



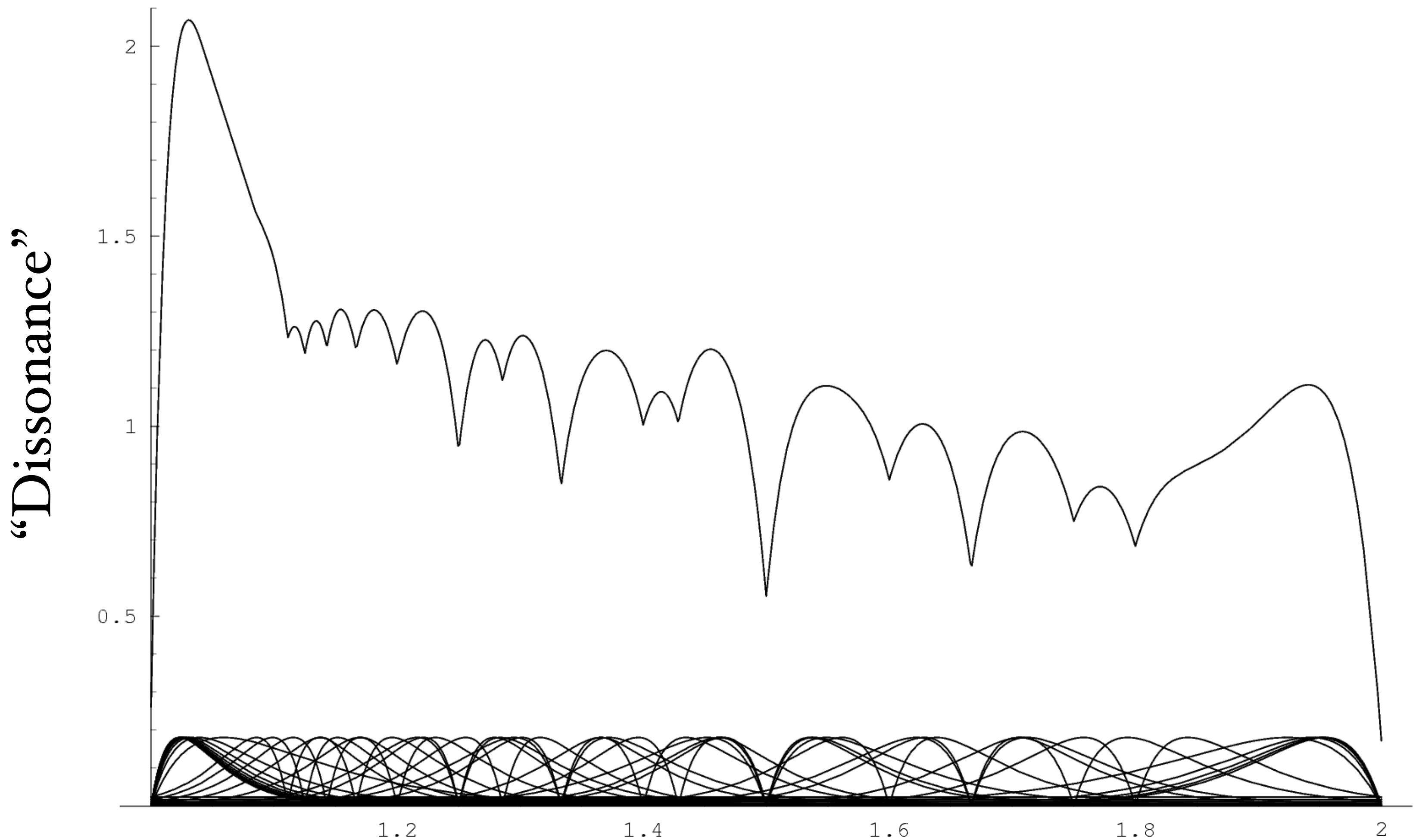
"Dissonance"

Empirical Psychoacoustics

Frequency *difference*

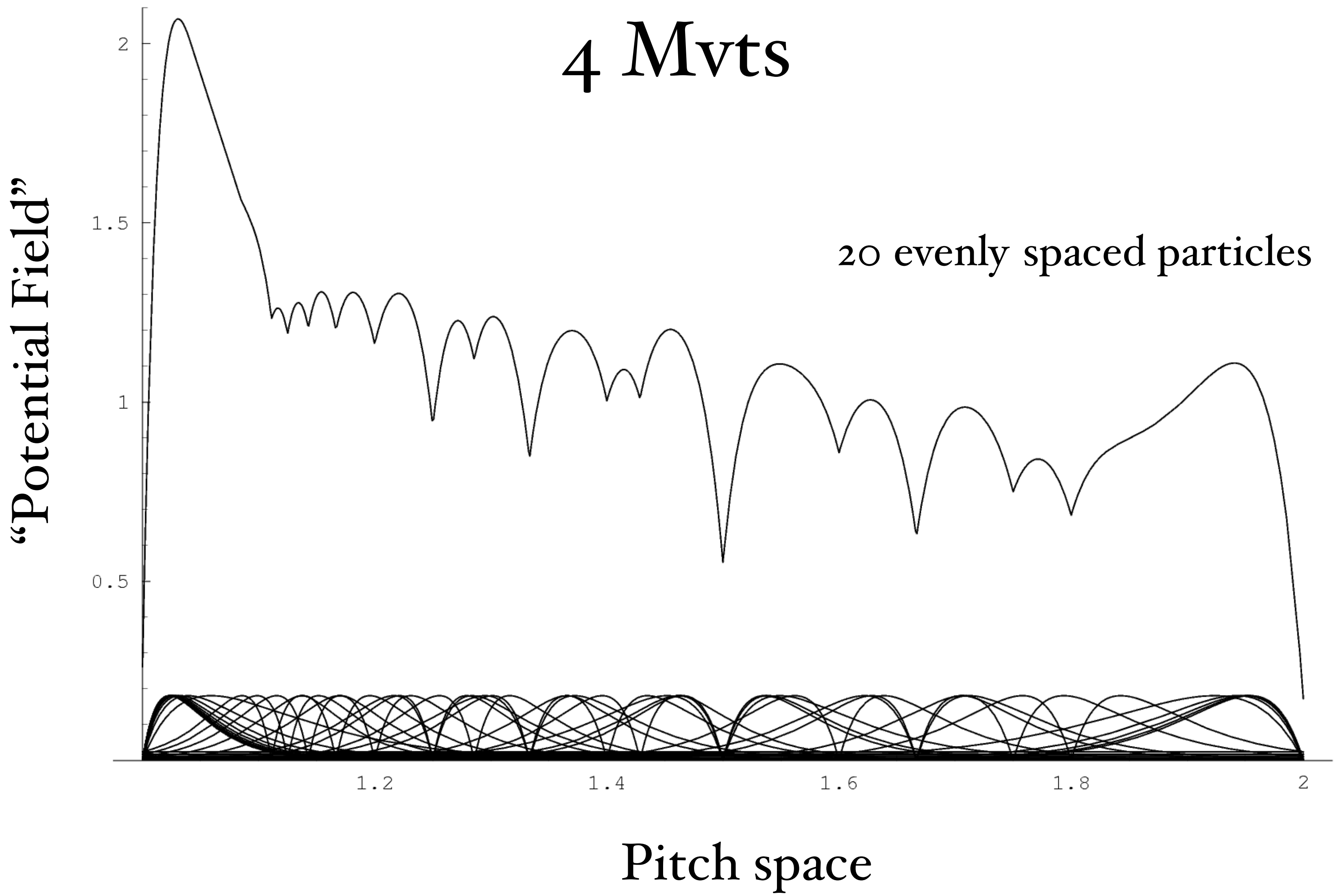
not ratio hence interval corresponding to maximum dissonance depends on *register*

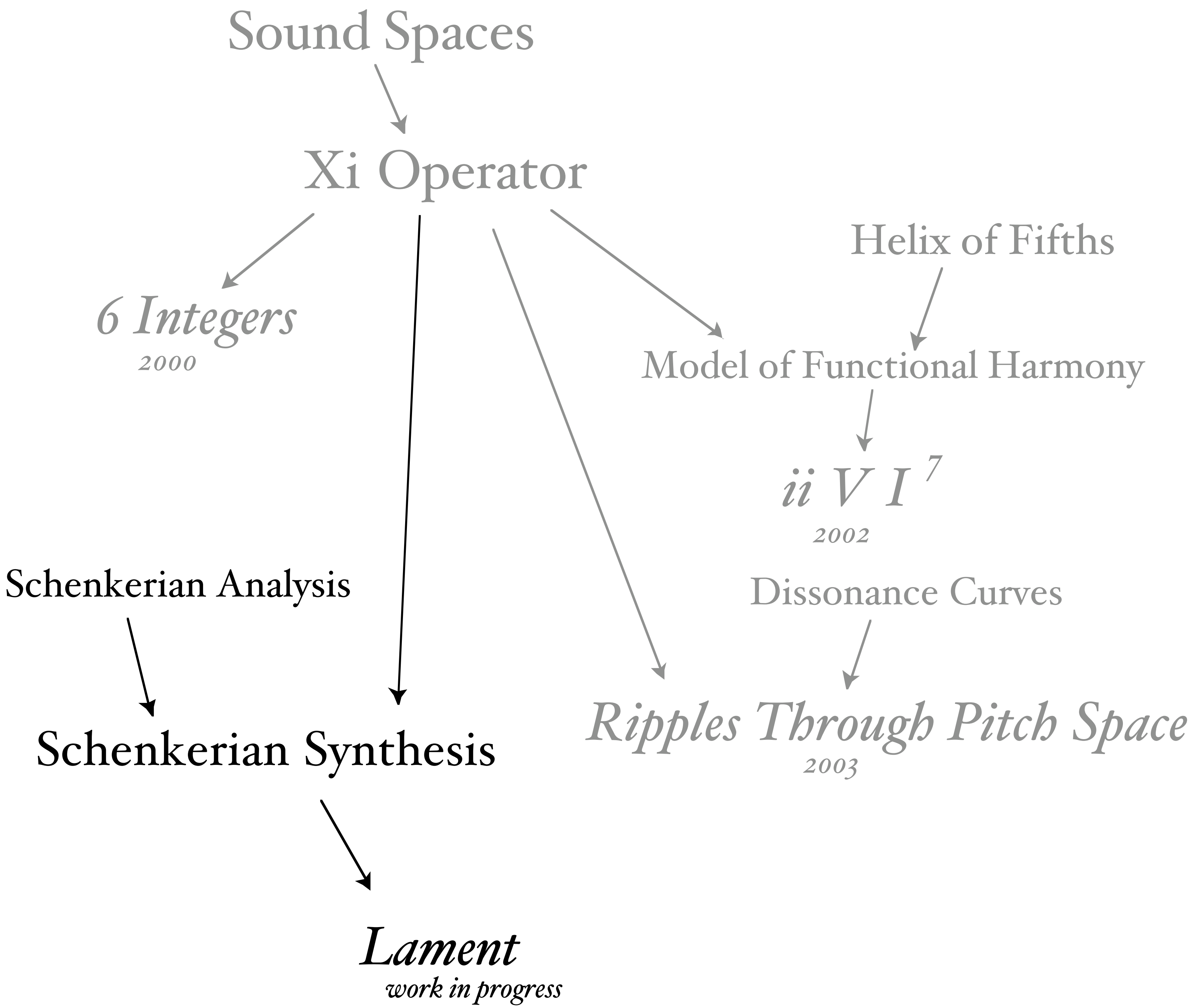
Dissonance of 2 *Harmonic* "*Buzzes*"



Frequency *ratio*
of 2 harmonic buzzes

Ripples Through Pitch Space 2003





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But what about melodies?

But what about melodies?

Idea: Do Schenkerian analysis in reverse via Xi   *Schenkerian synthesis!*

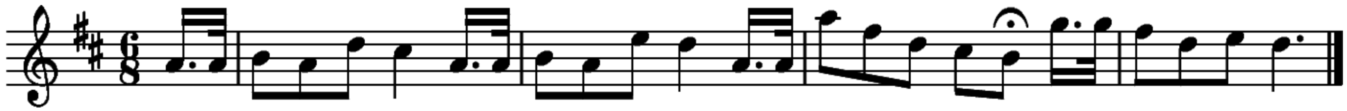
But what about melodies?

Idea: Do Schenkerian analysis in reverse via Xi *Schenkerian synthesis!*

But what is Schenkerian analysis?

Introduction to Schenkerian Analysis in One Page!

Introduction to Schenkerian Analysis in One Page!

Happy Birthday!

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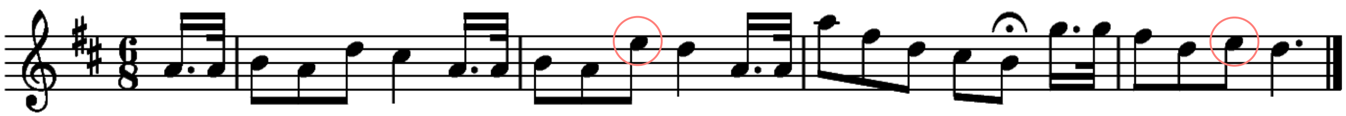
Happy Birthday!

Relative structural significance?

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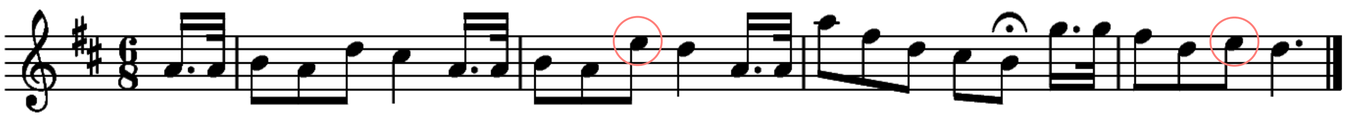
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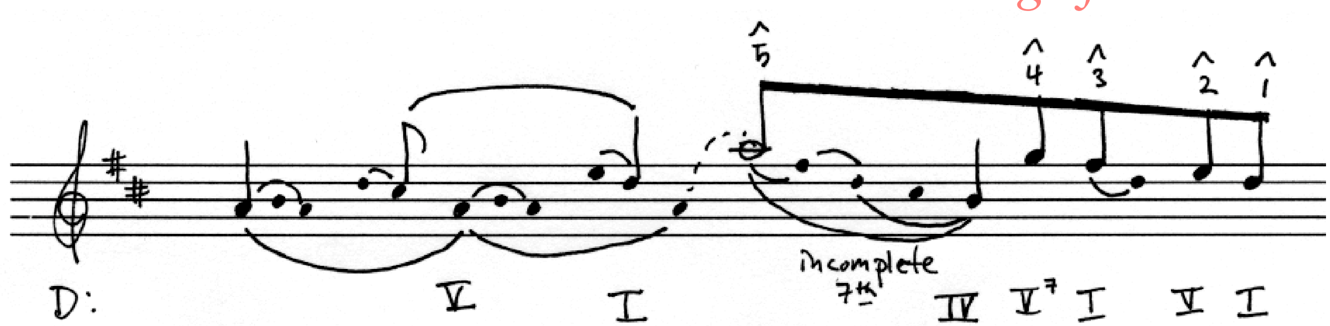

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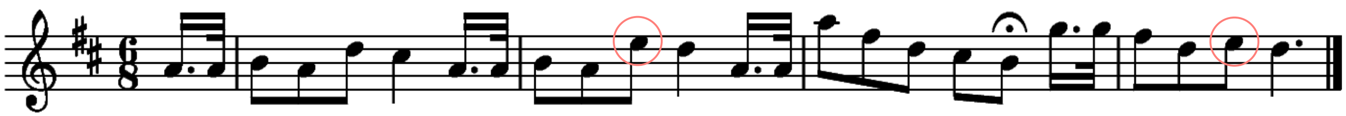
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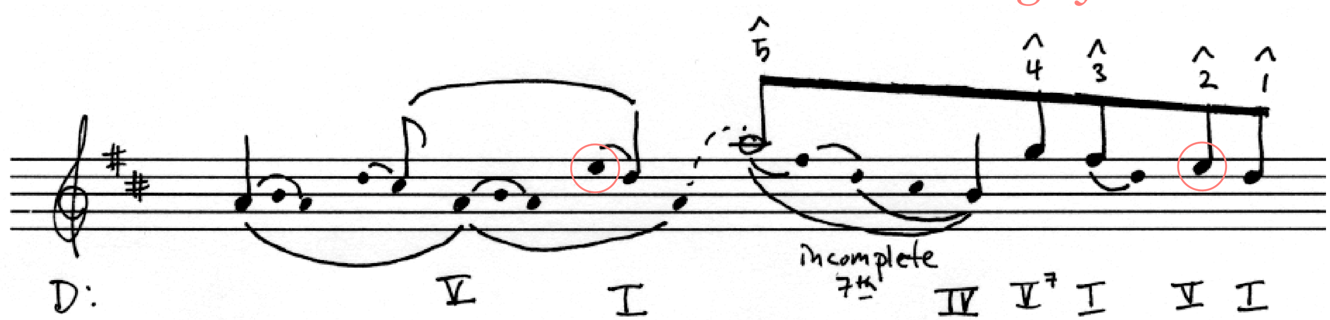

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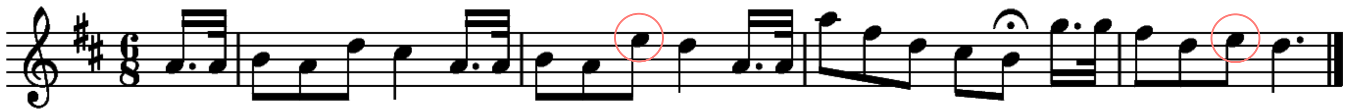
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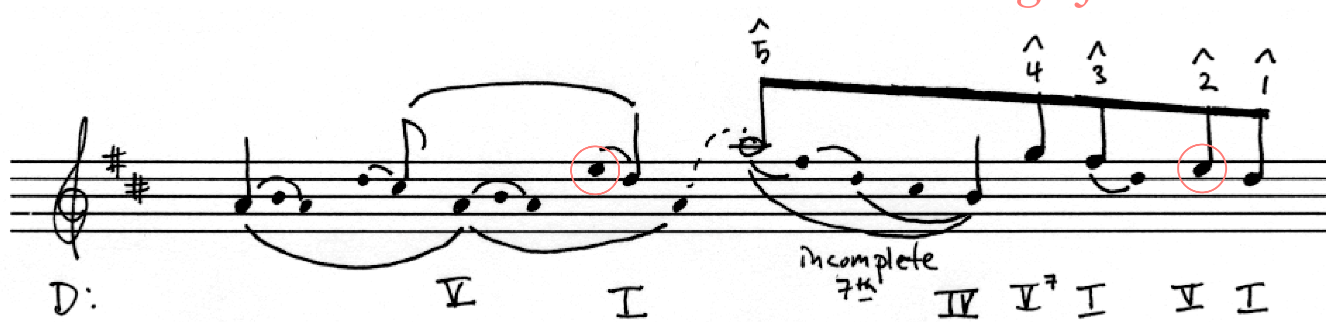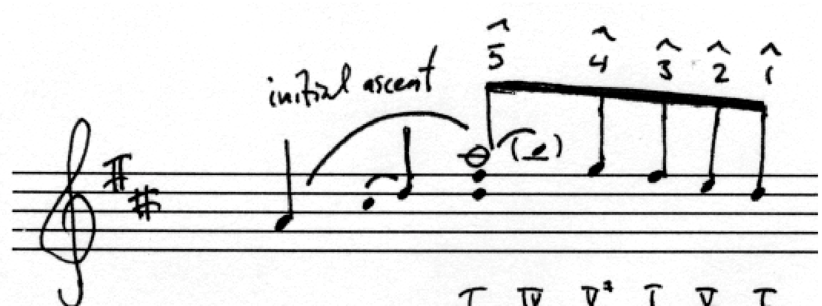

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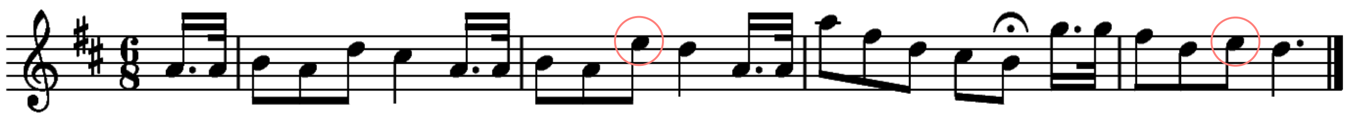
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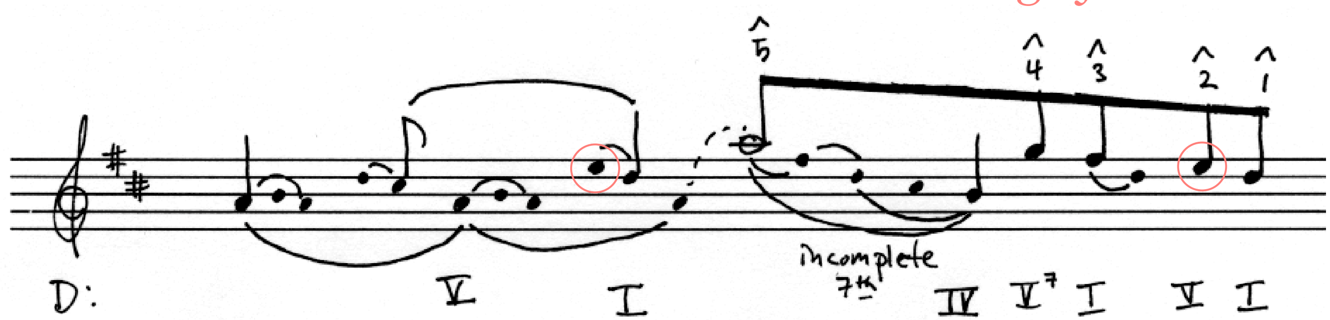
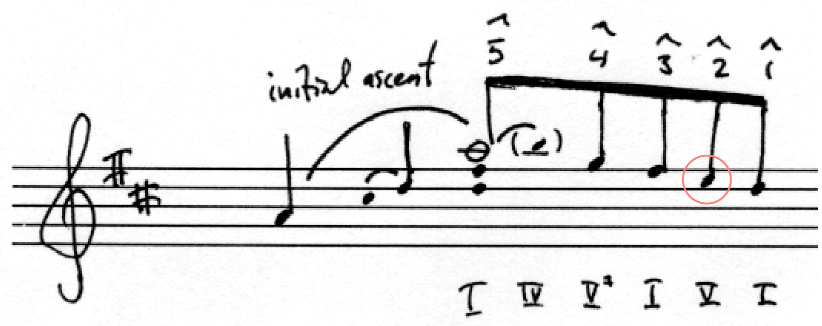
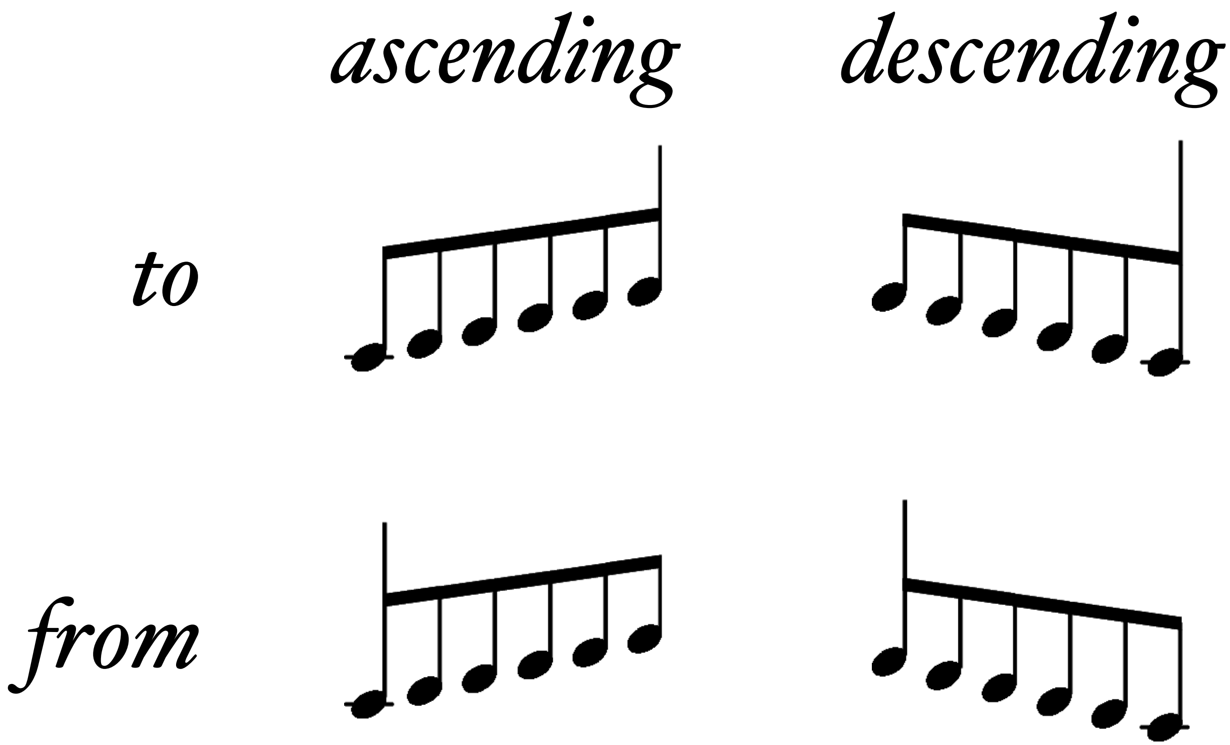




Inheritance Functions for Schenkerian Synthesis



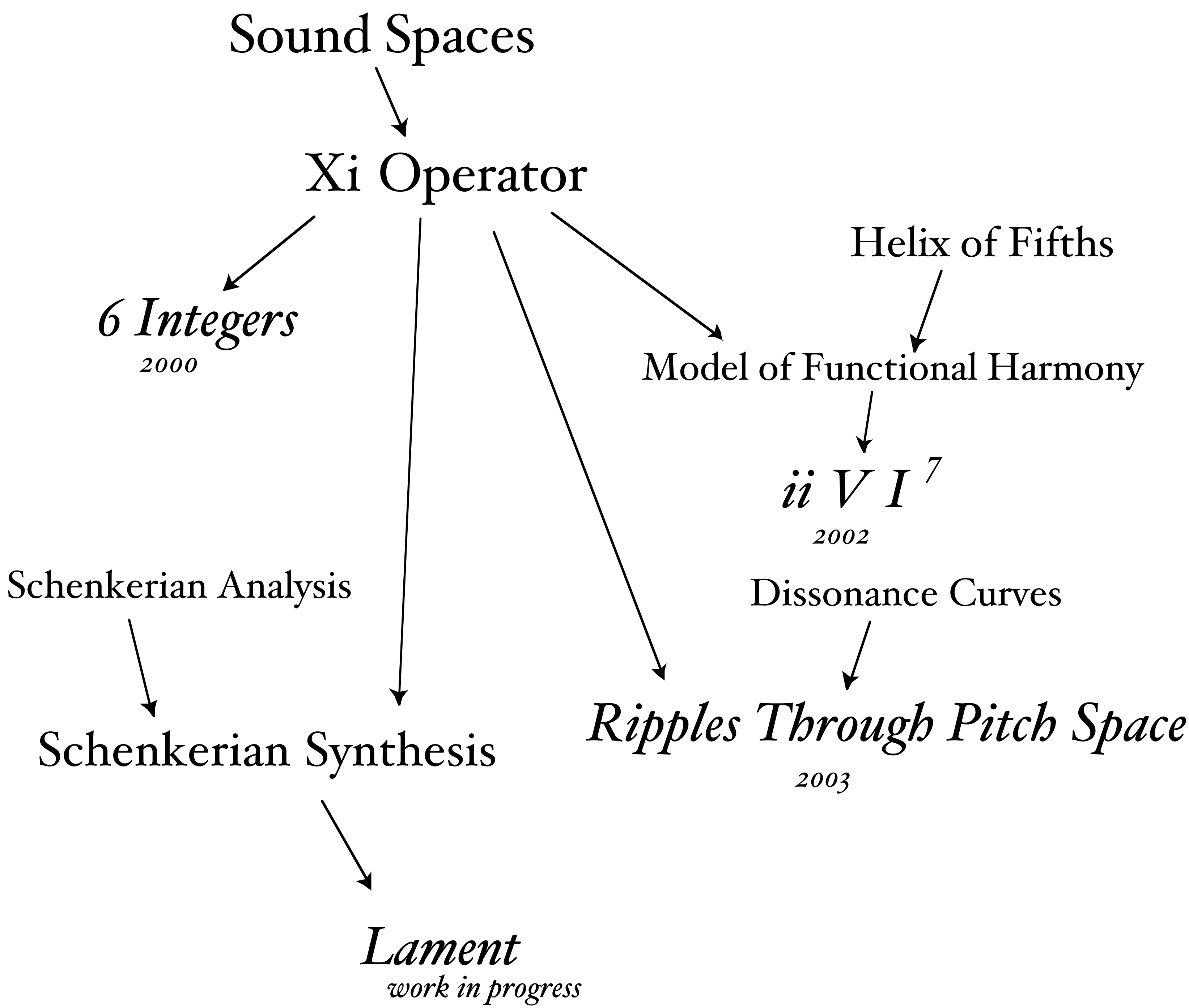
Lament

work in progress

“Lyre from Ur”
from ca. 2400 B.C.
Source: Oriental Institute

2 Mvts
so far

Melodic line created with Schenkerian Synthesis: embedded within self.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Summary:

Mathematical structures were described which can be used to produce music through computation. Most important was the versatile Xi Operator, which may be used to construct models for expressive rhythm, functional harmony and melody.

Please visit my web page

www.mctague.org/carl

to hear these pieces and others.

Want the mathematical structures to be *musically meaningful* whatever that means at least inspired or informed by musical experience, intuition or theory.

Can also use $0,1 \to \mathbb{R}$ to control continuous parameters of sound.

E.g. loudness

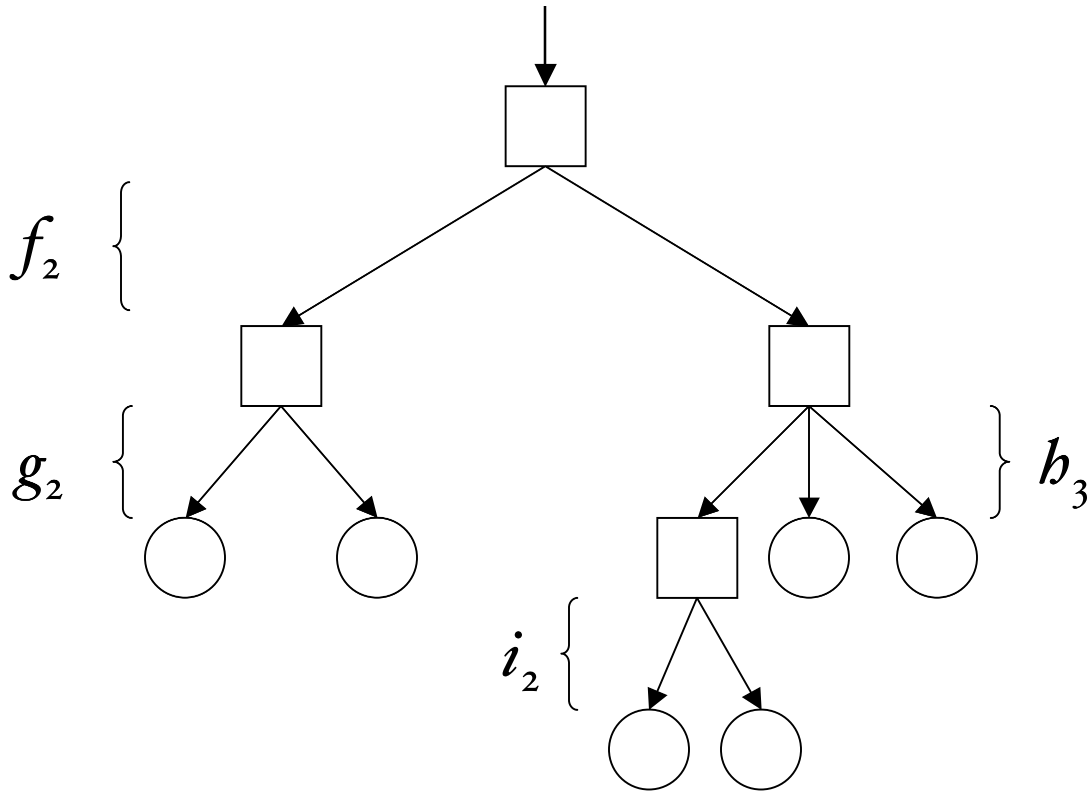
I call this construction the *Xi Operator* Ξ

Given a family of inheritance functions and an ordered list of sound spaces, it produces a new sound space:

$$\begin{array}{c} \left\{ f_n : A \to B^n \right\}_n \\ \left(c_i : B \to (L^p)^m \right)_{i=1,\ldots N} \end{array} \xrightarrow{\ \Xi\ } A \to (L^p)^m$$

An alternate view;

inductive use of Xi as information propagating through a tree:



Information flows down the tree, manipulated at each branch by the local inheritance function until it reaches the Os, which denote possibly distinct, existing sound spaces.